Tree-walking automata

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Plan

-A tree-walking automaton

-Expressive power

-Pebble automata and logic

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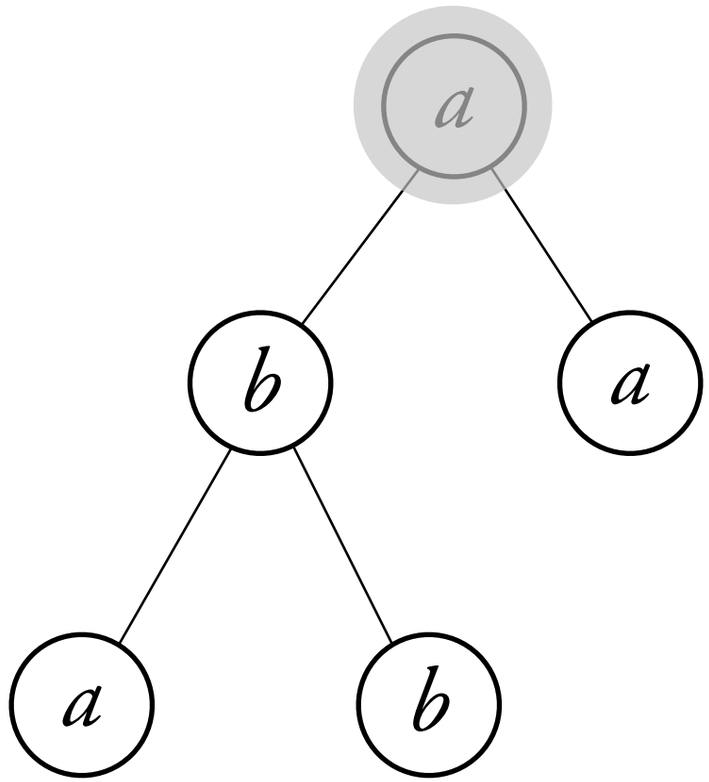
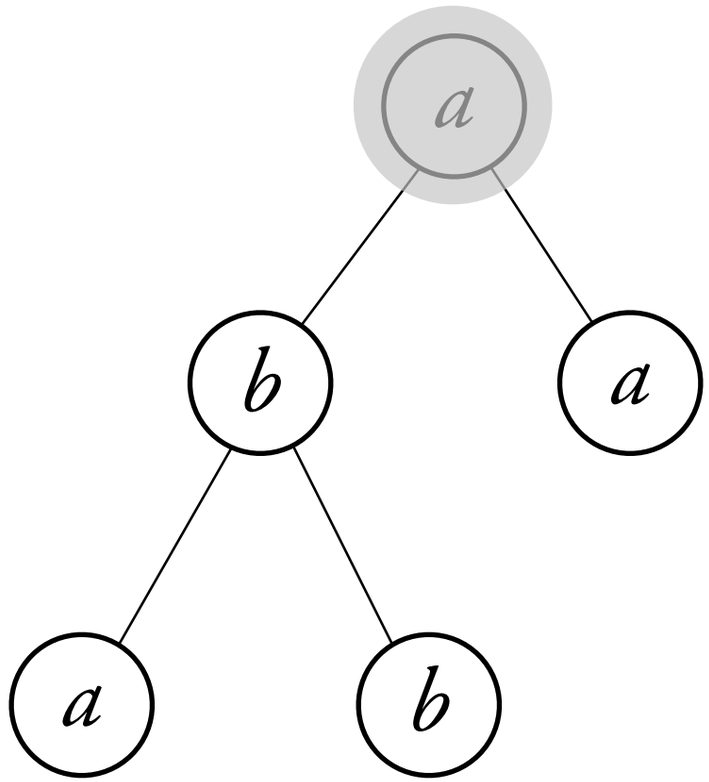
Trees are finite, binary and labeled



Trees are finite, binary and labeled

A tree-walking automaton is sequential and two-way.



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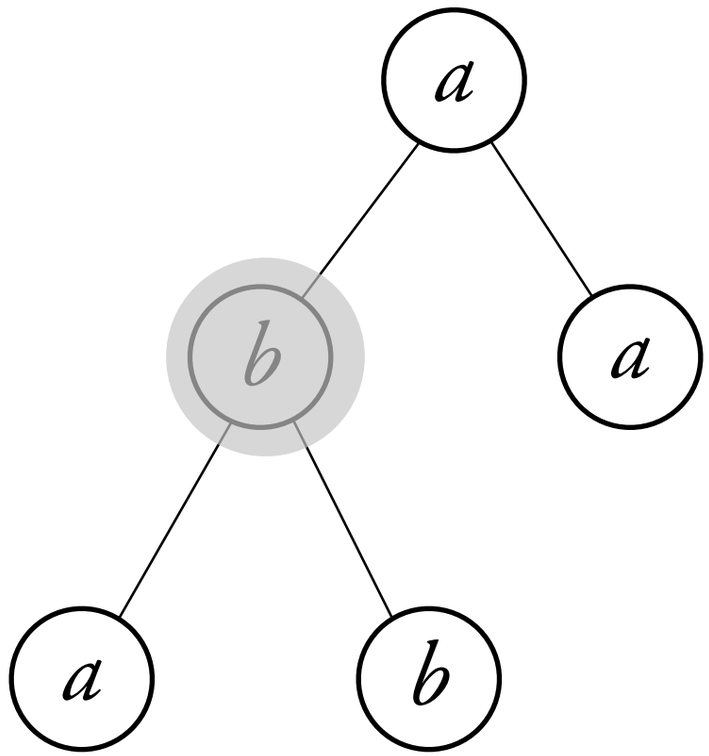

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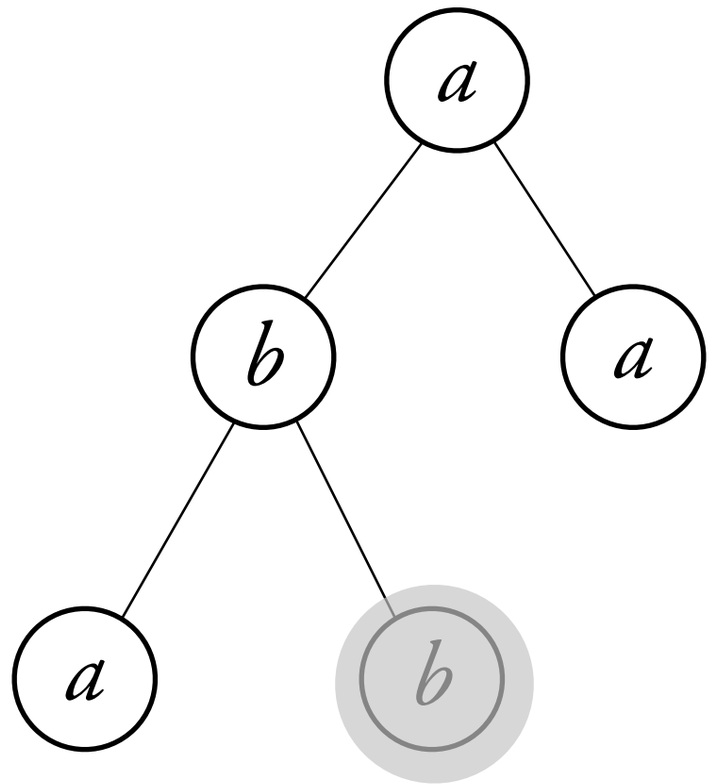

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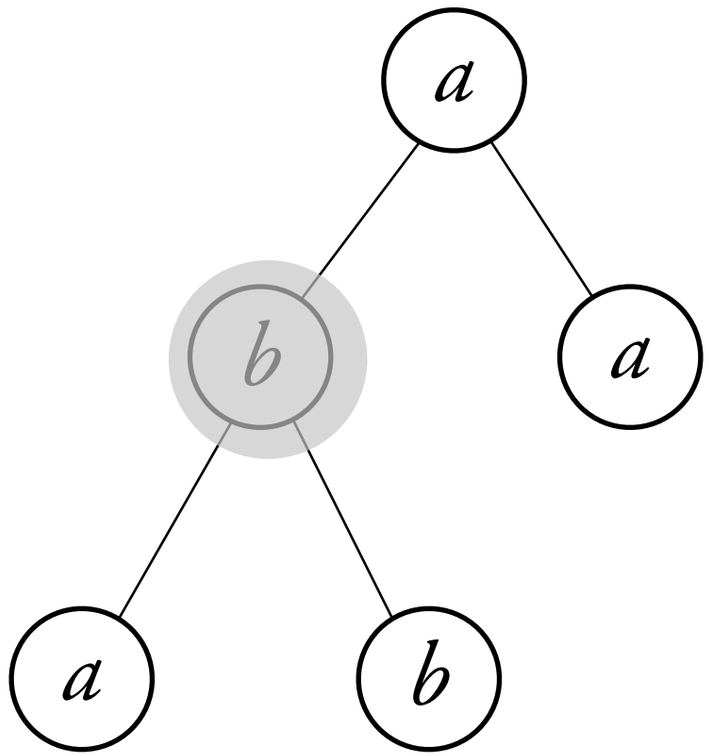

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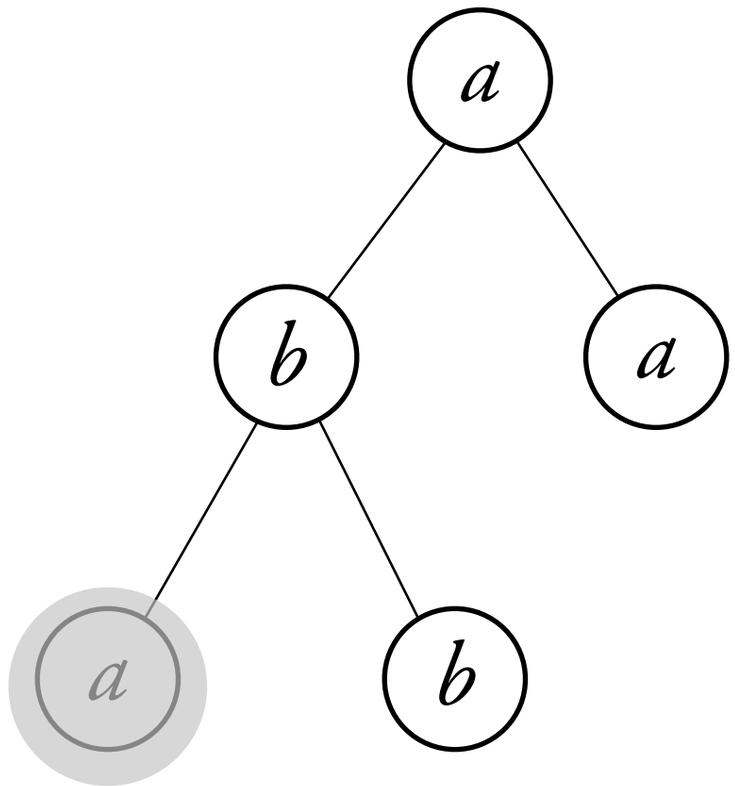
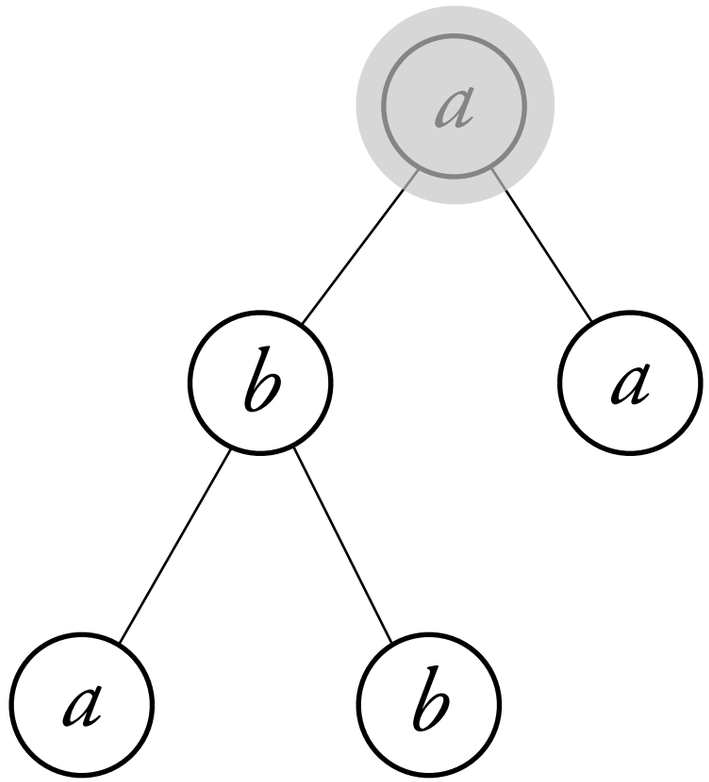


If the state is p and the node is the root with label a, then move to the left child and change state to q.



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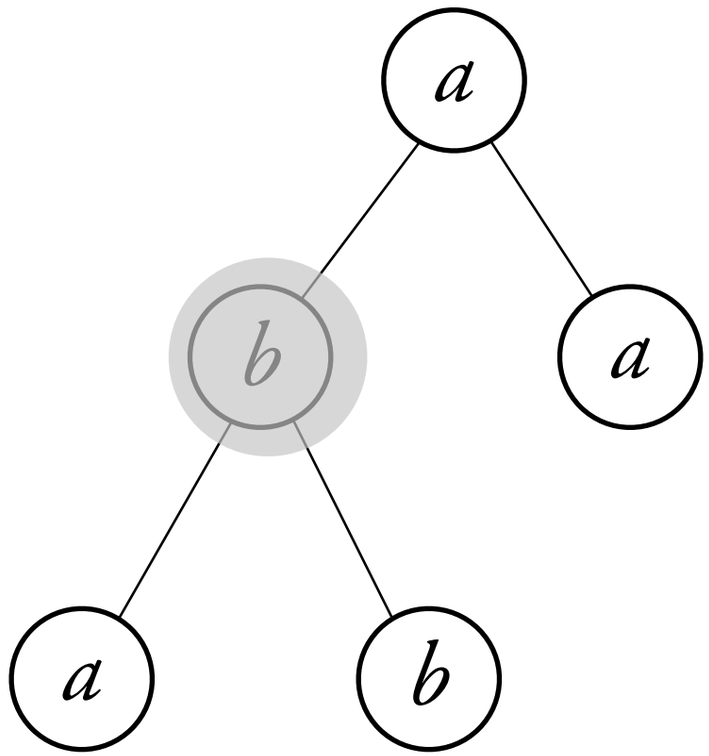

test

If the state is p and the node is the root with label a,
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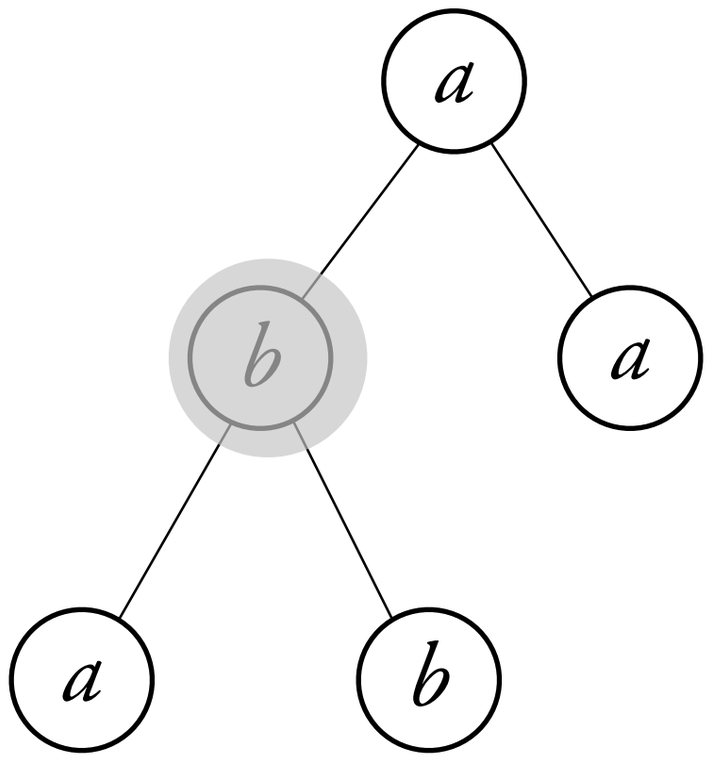
command



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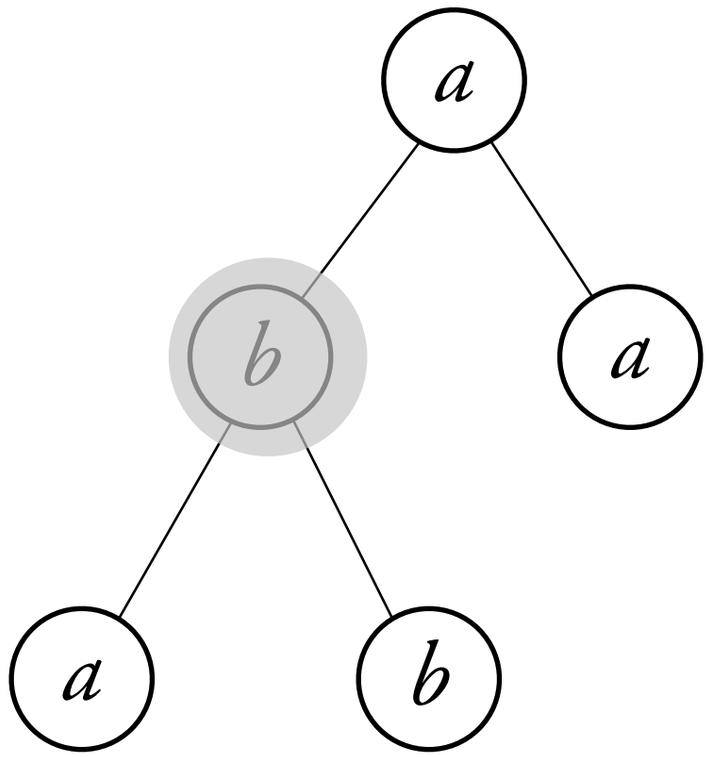
Tests are boolean combinations of:
has label a,
is right/left child,
is leaf



Commands are:
move left/right/up,
accept,
reject

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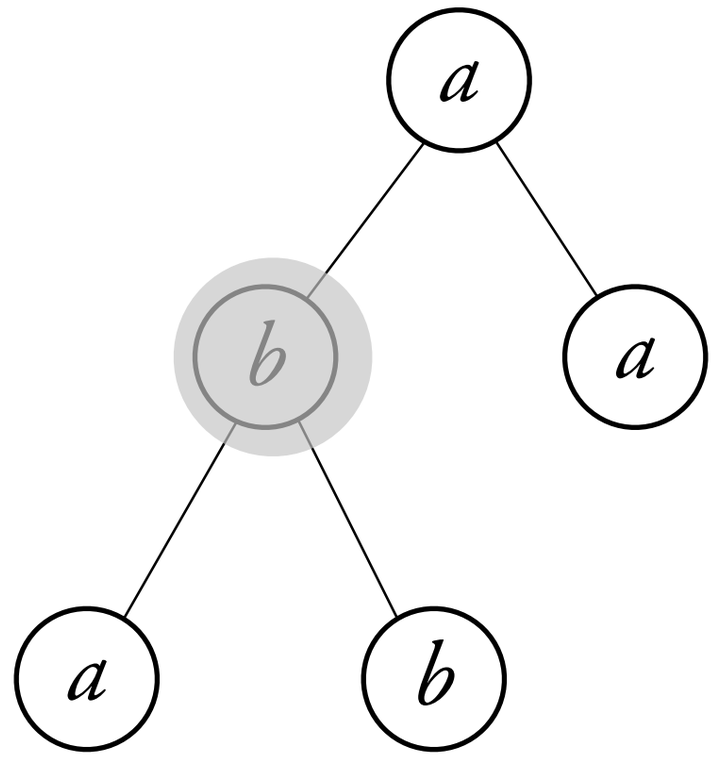

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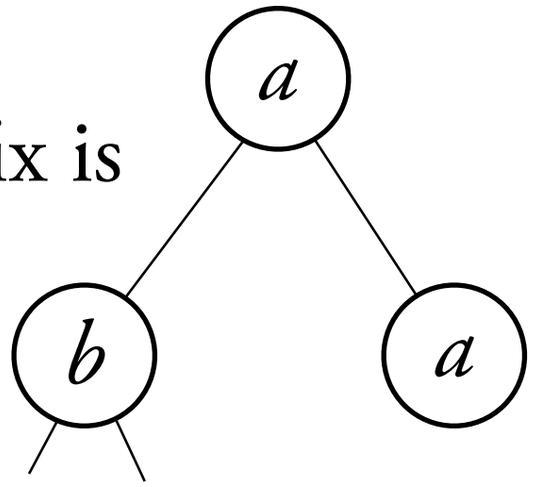
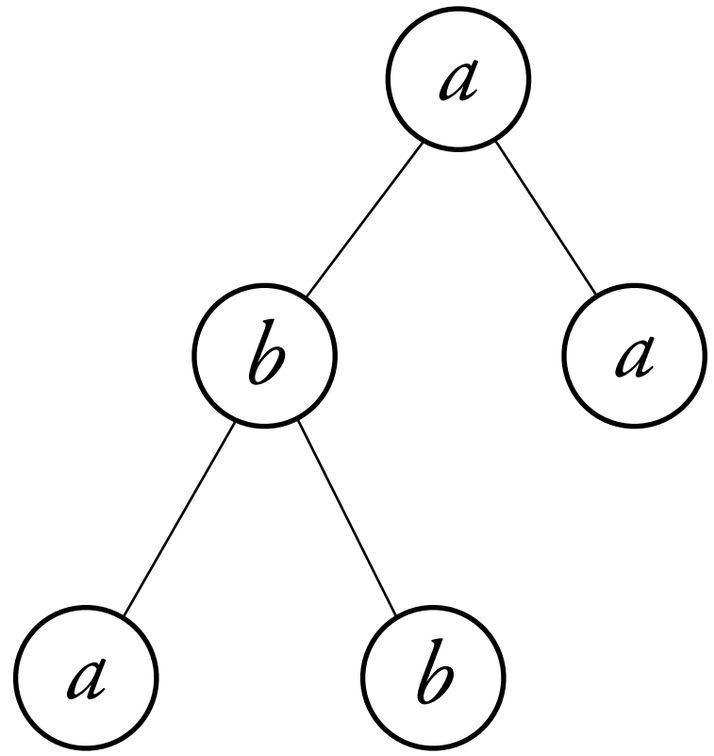
Def. A tree walking-automaton is a tuple $\langle Q, q_I, \Sigma, \delta \rangle$

states — initial state — alphabet — transitions

Example.
Check if the prefix is





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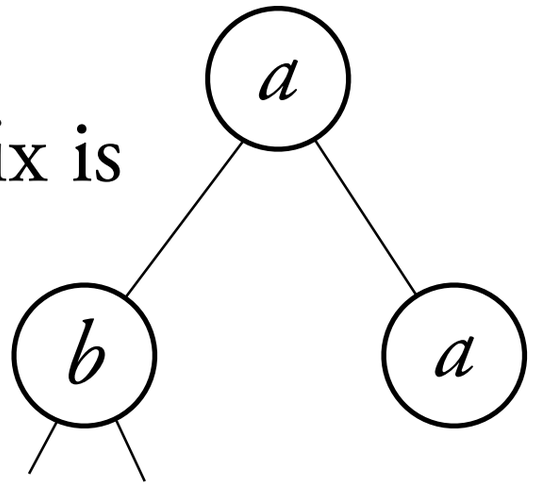

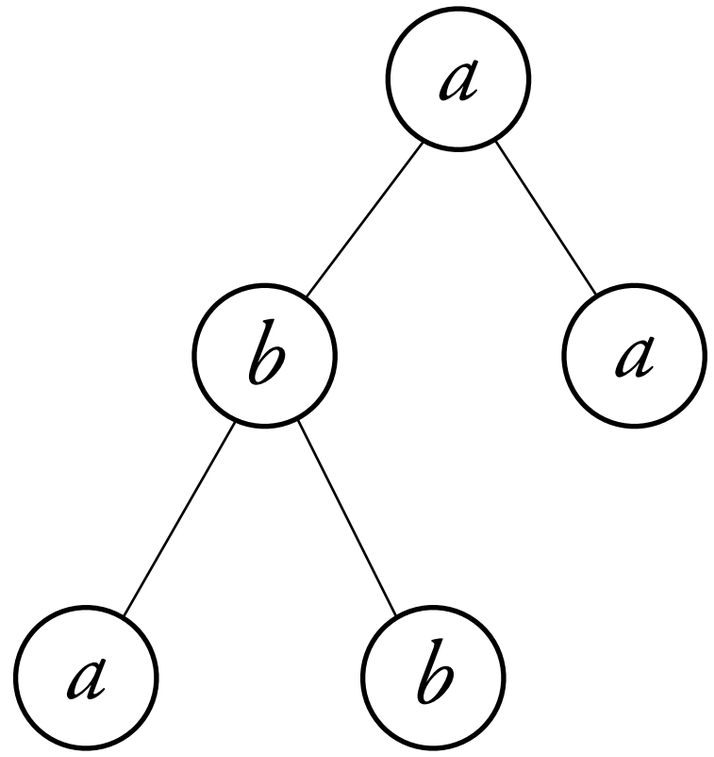

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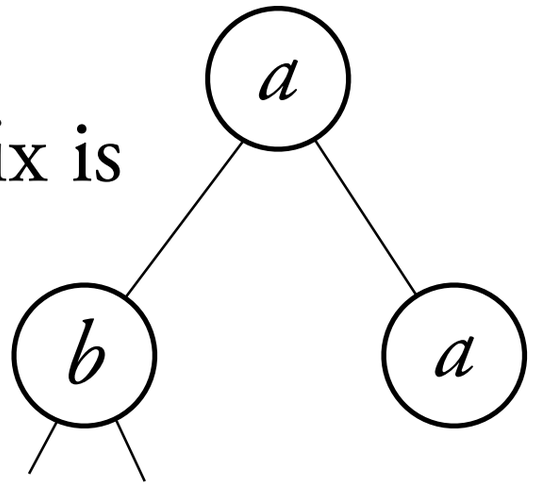

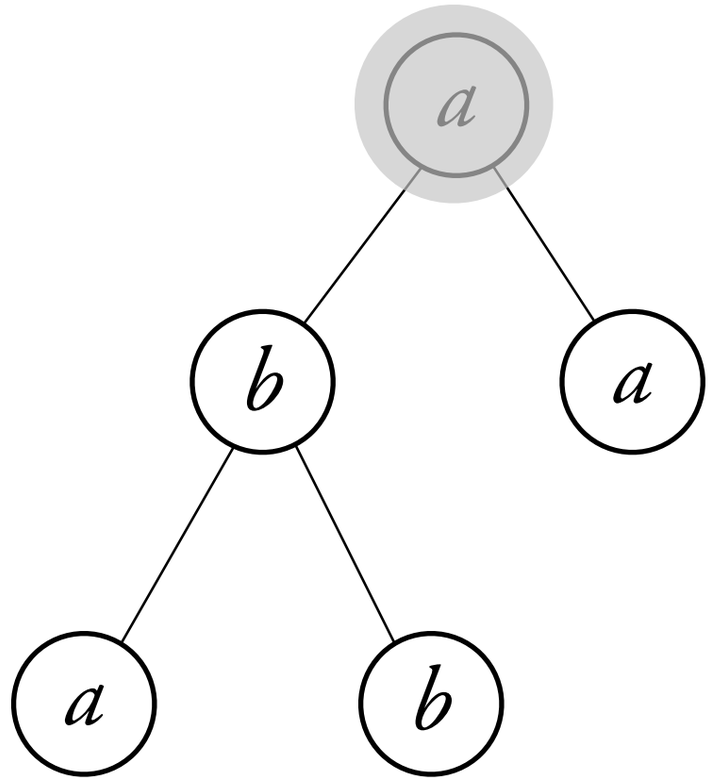

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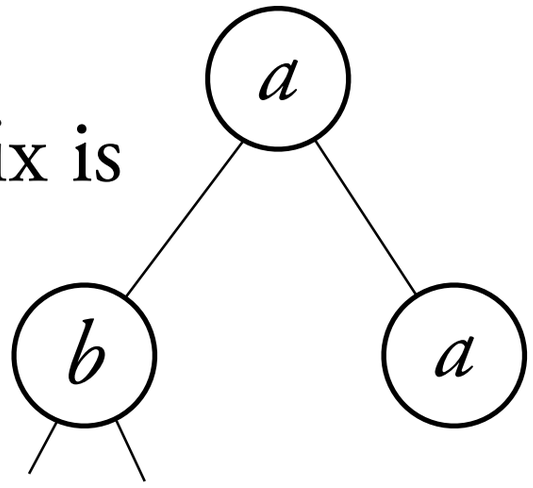

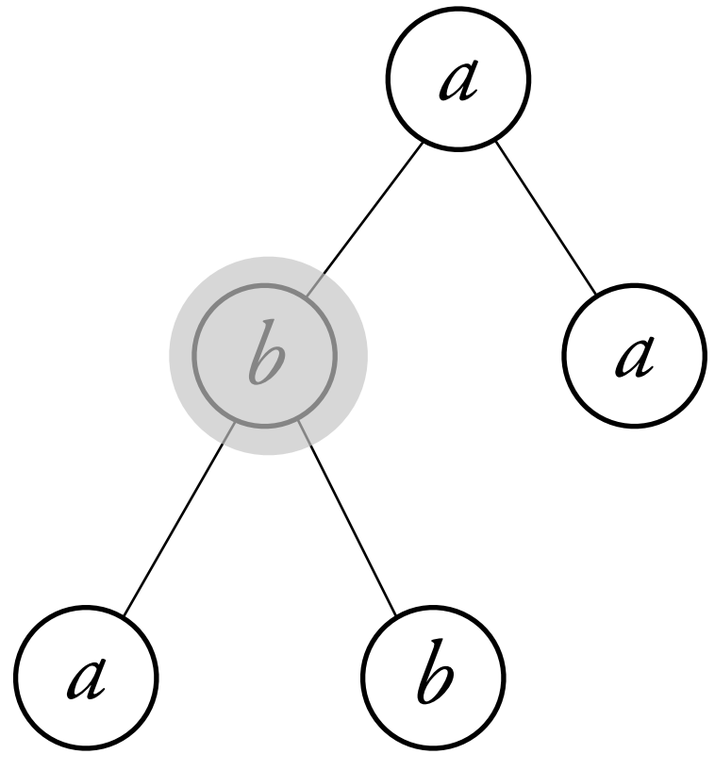

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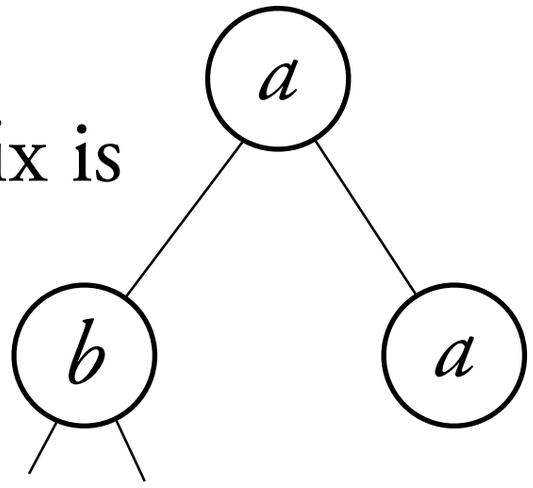

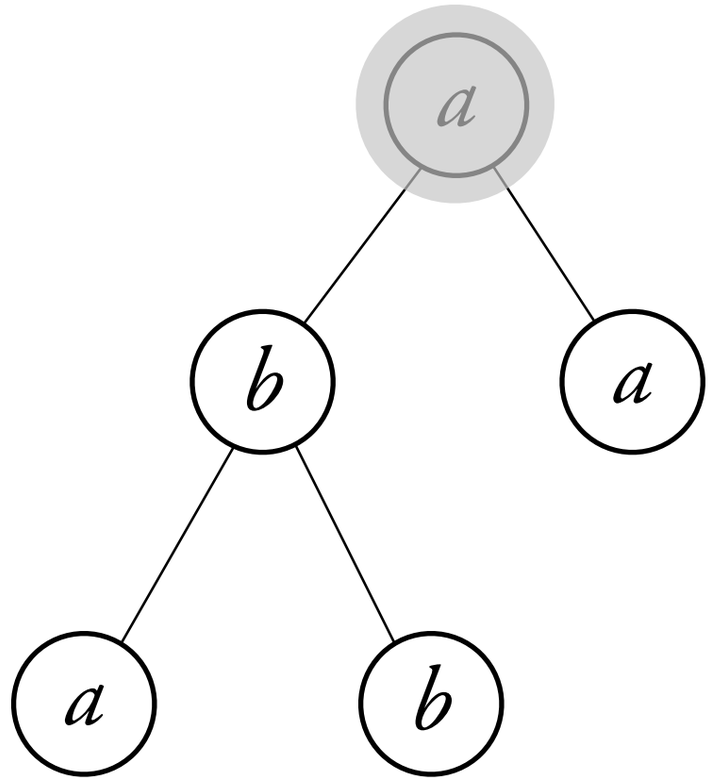

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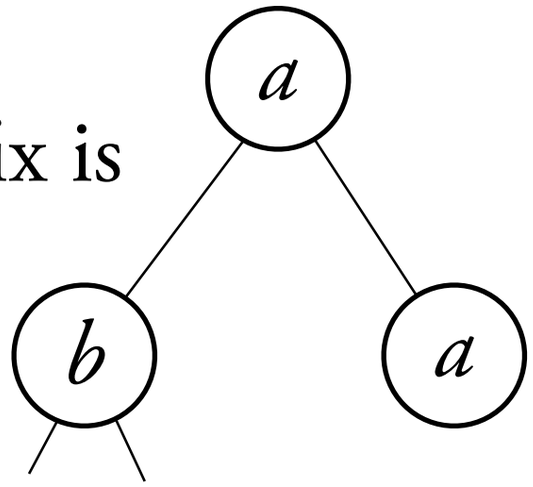

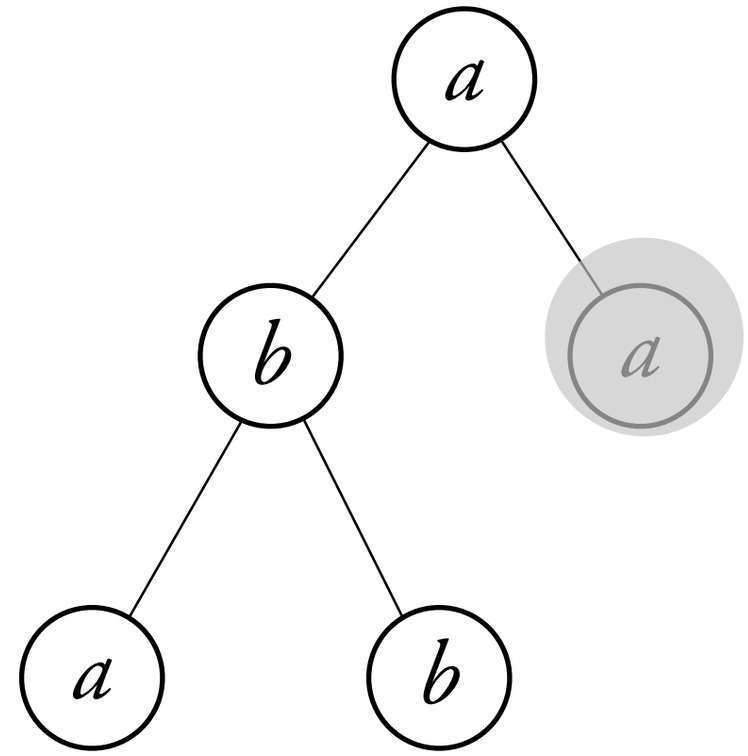

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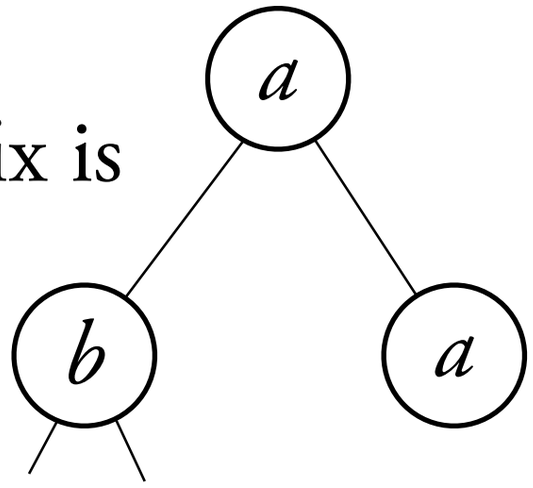

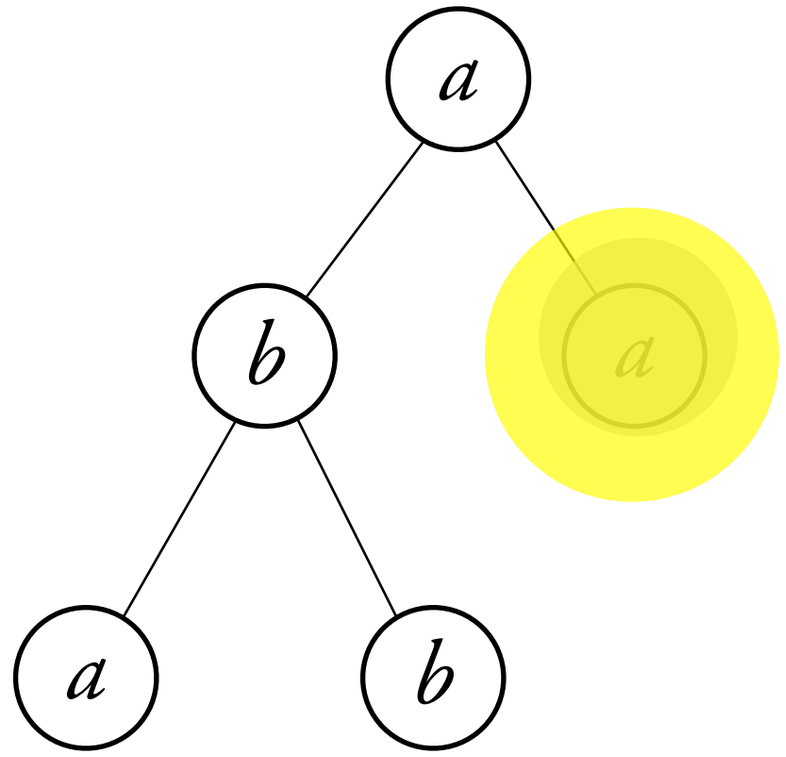

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In state *p*, move left.
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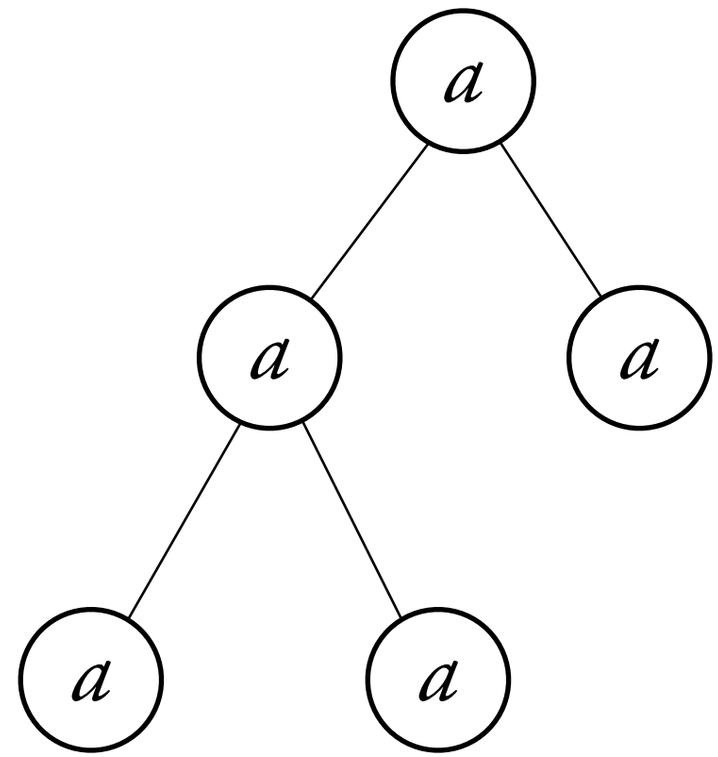

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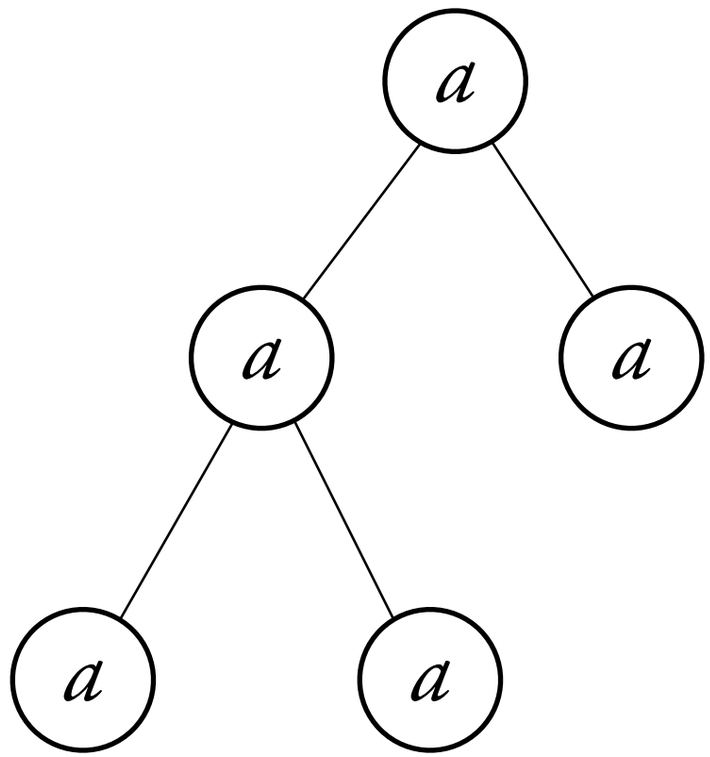

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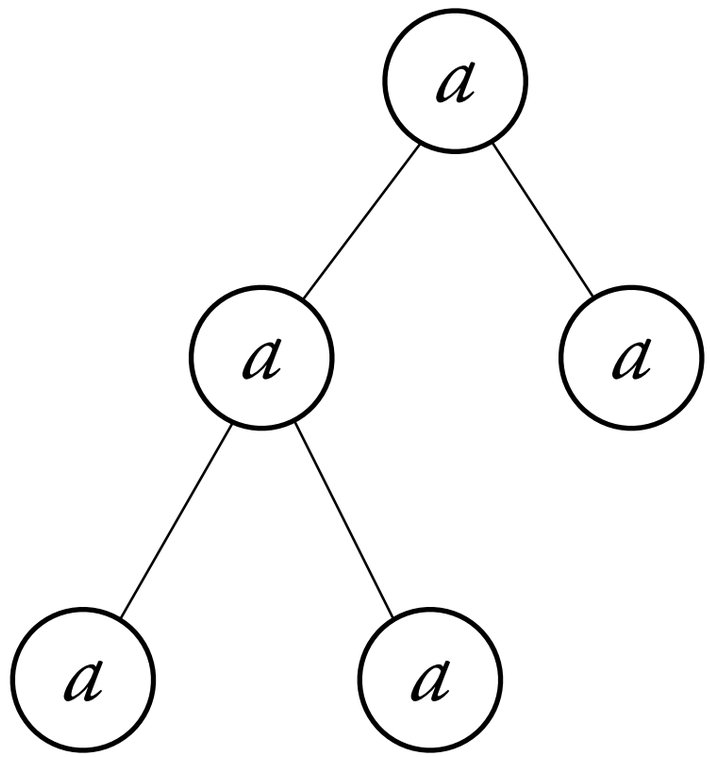
fromup, fromleft, fromright



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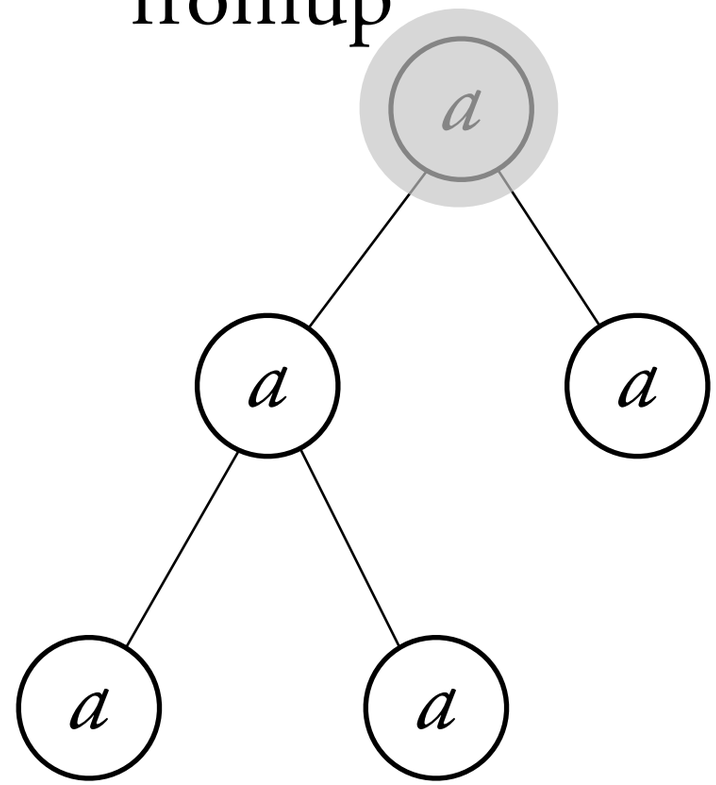

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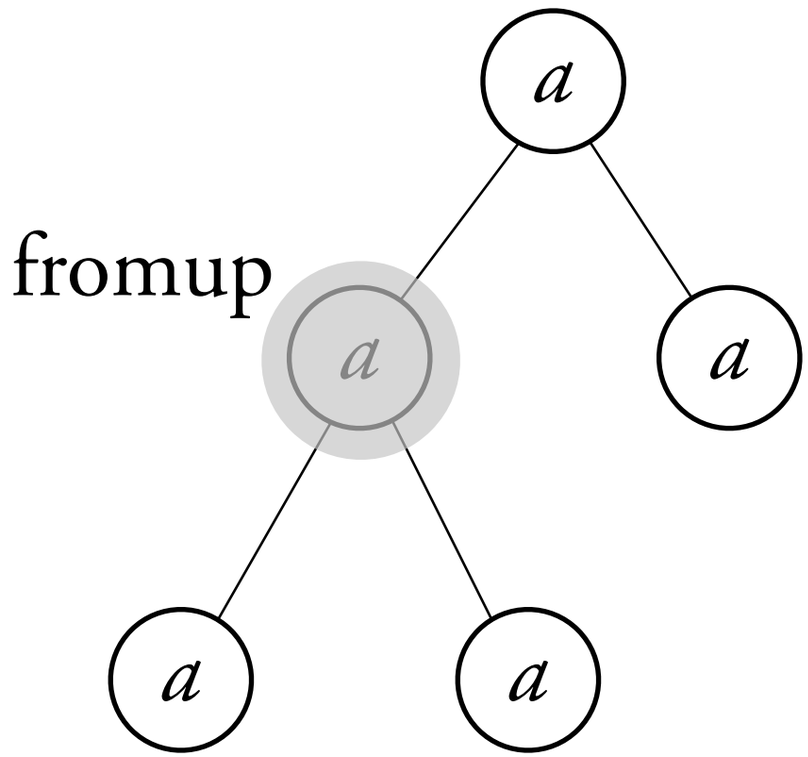

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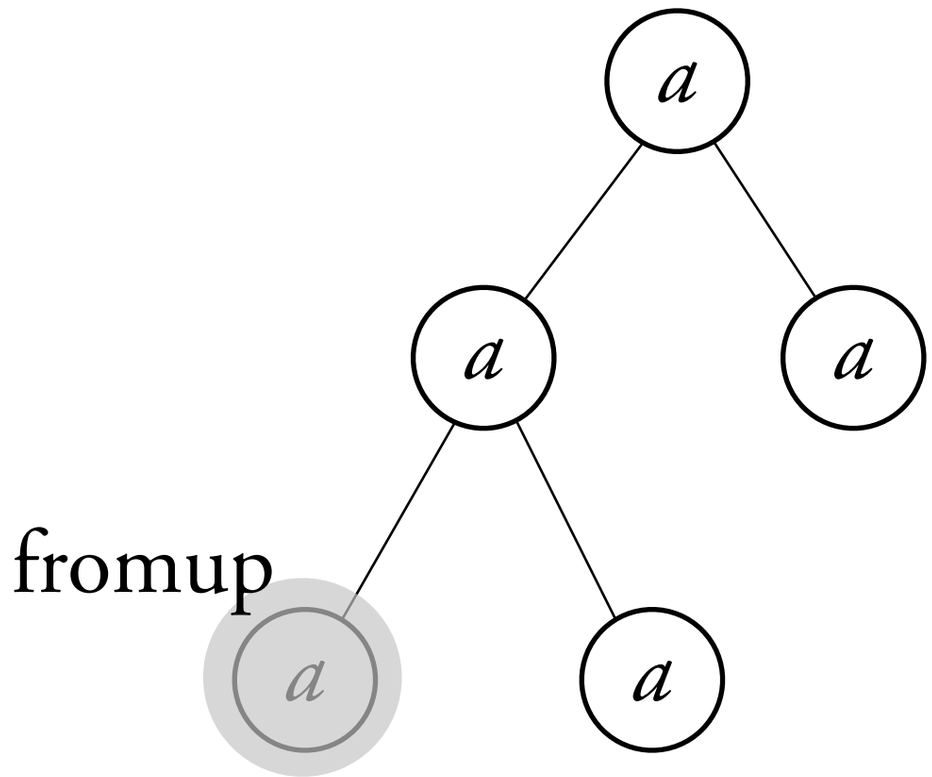

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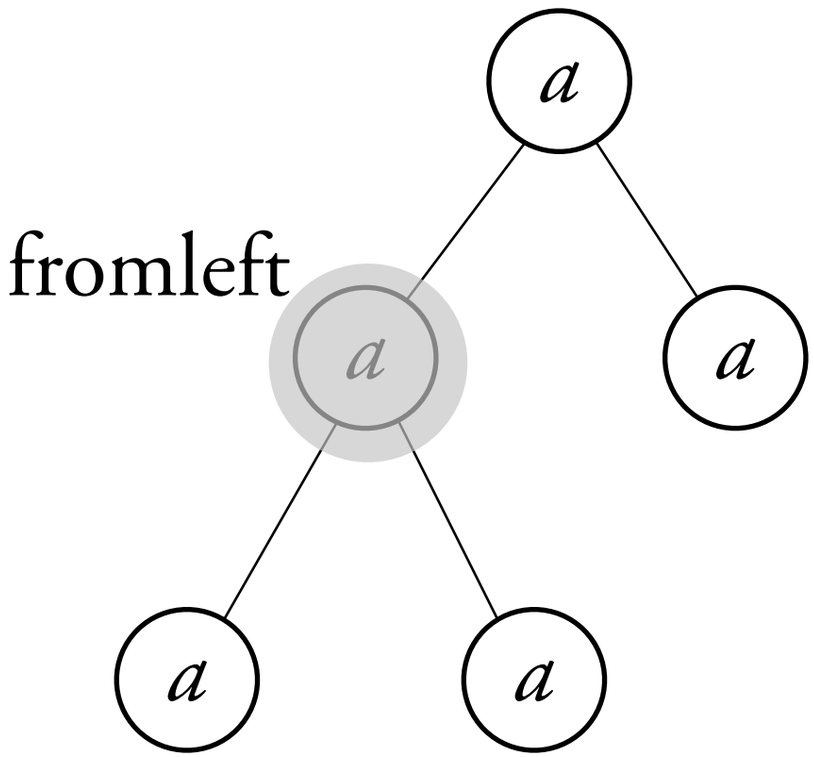

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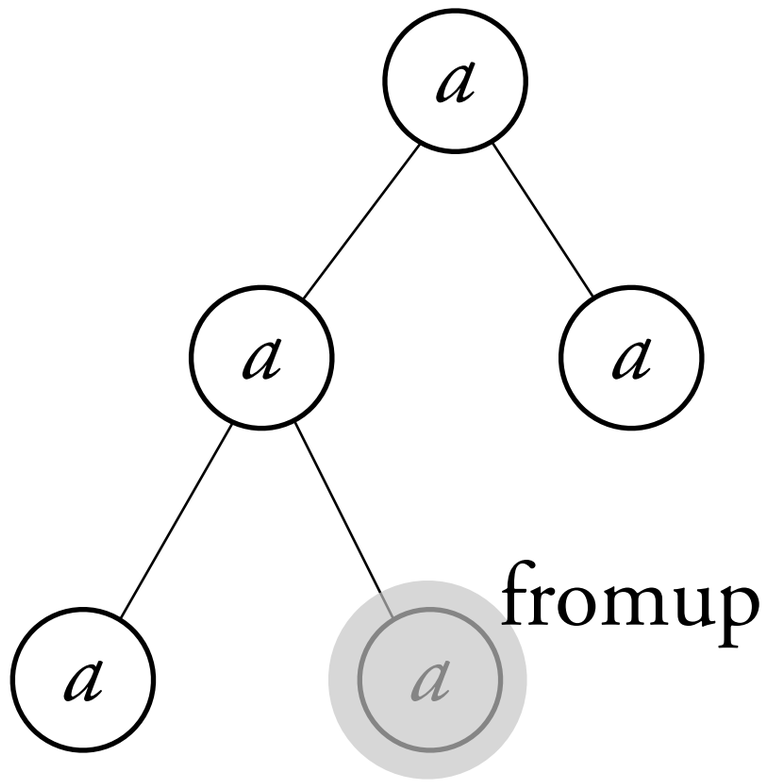

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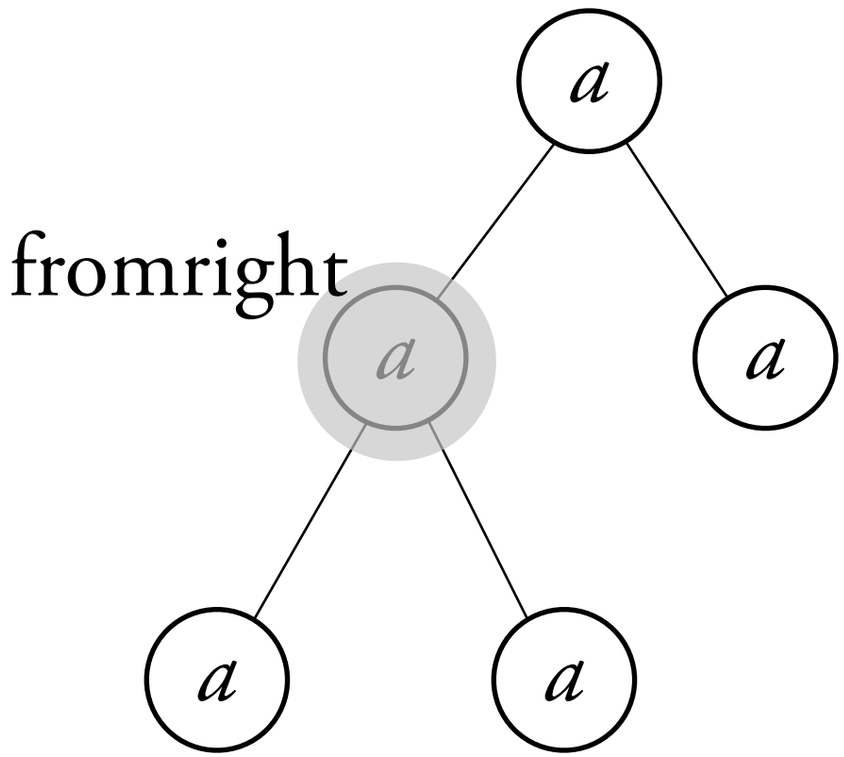

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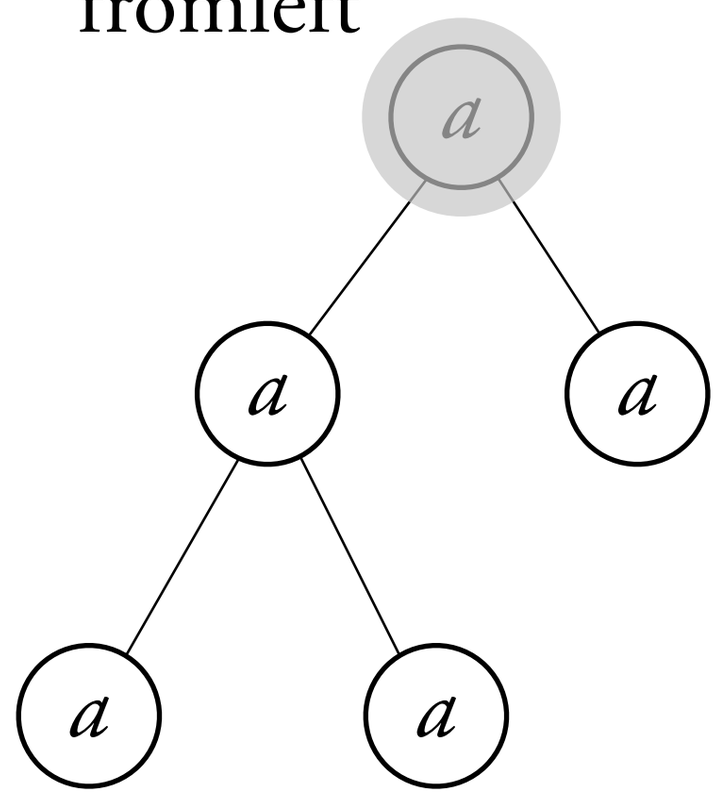

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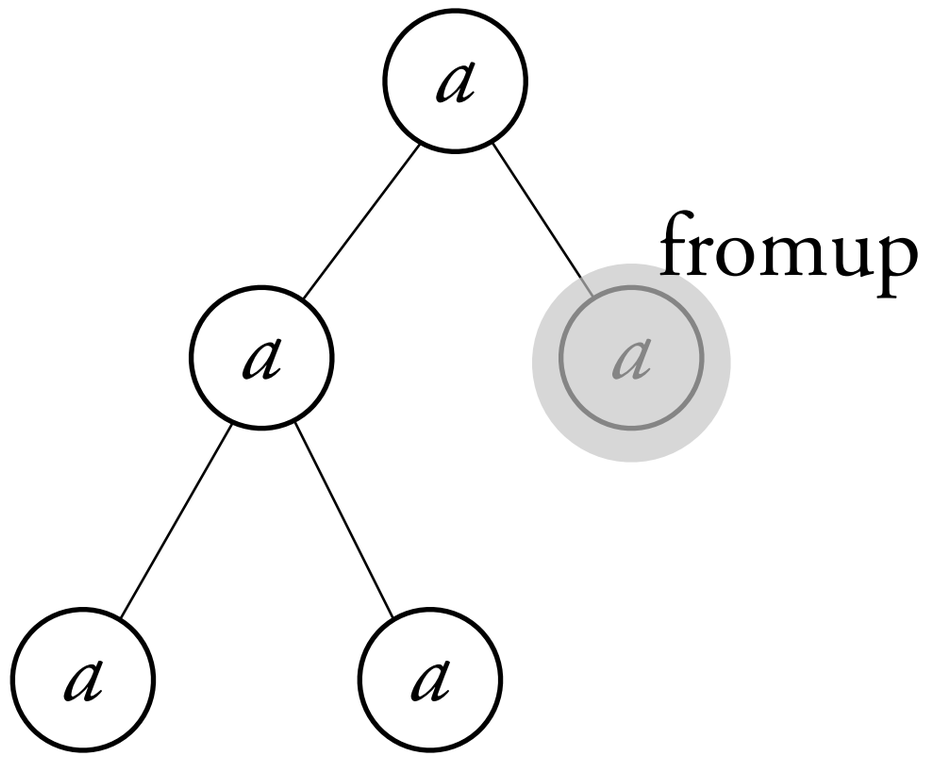

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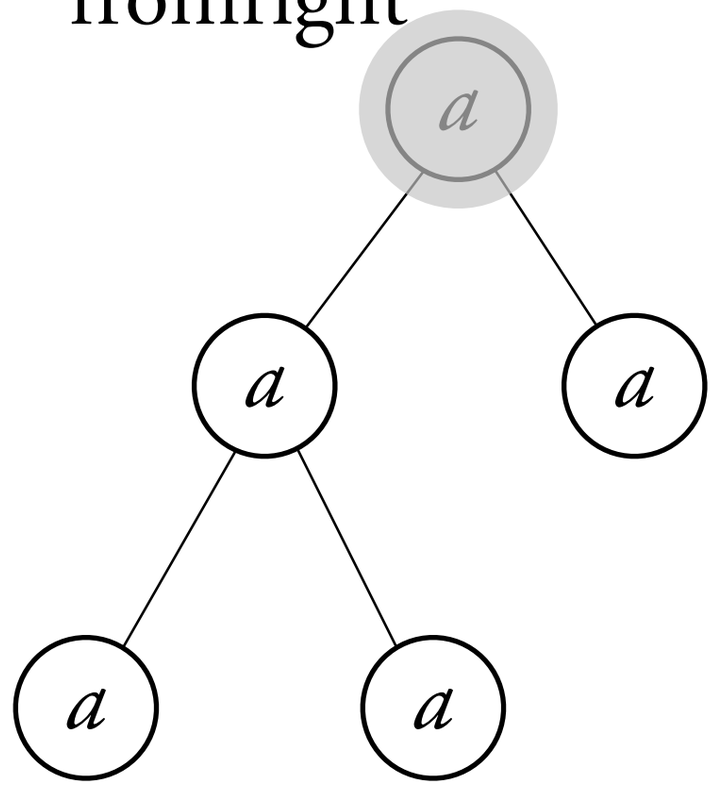

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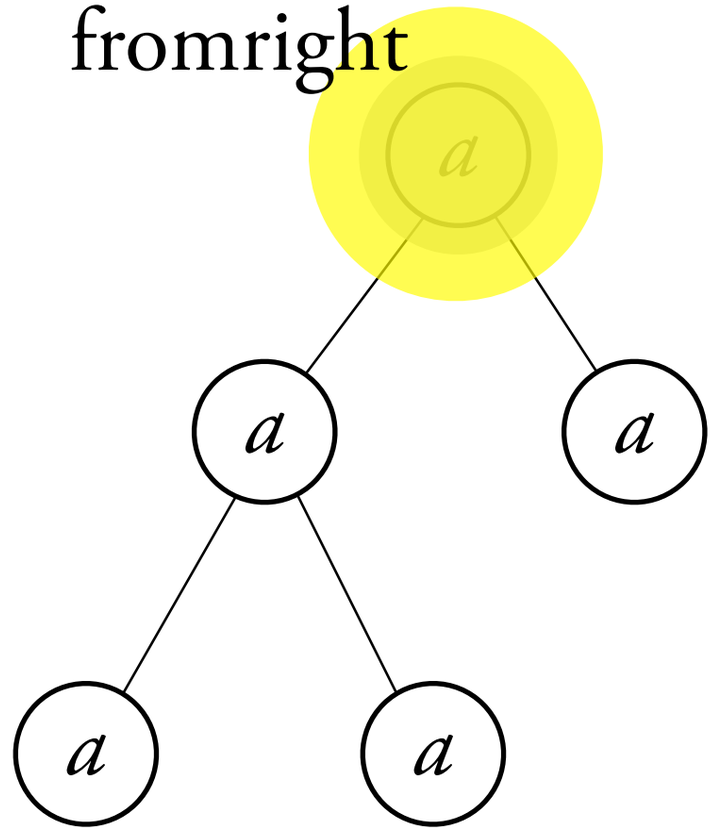

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Complemenation is difficult!

Open problem:
Are nondeterministic tree-walking automata closed under complementation?

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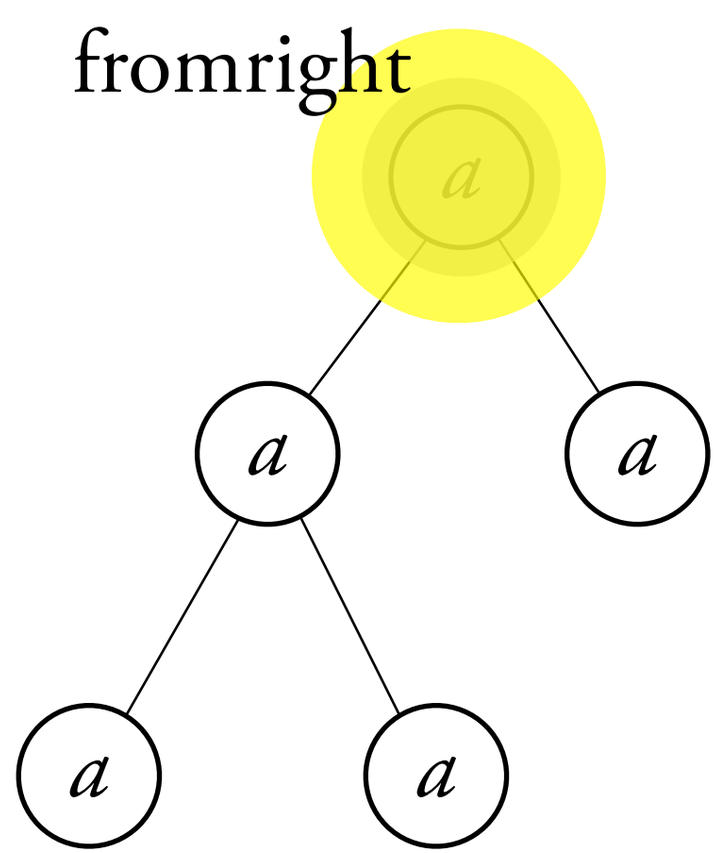
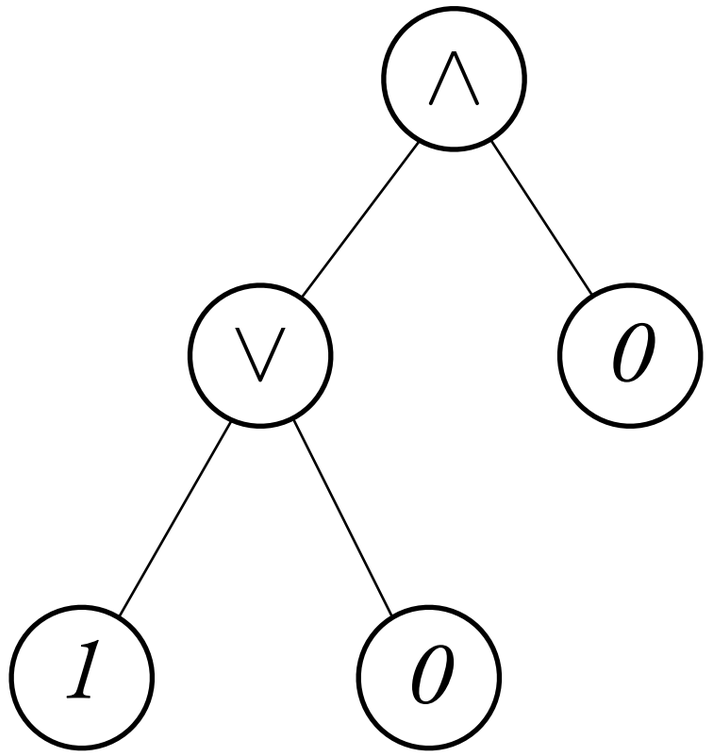


A clever tree-walking automaton



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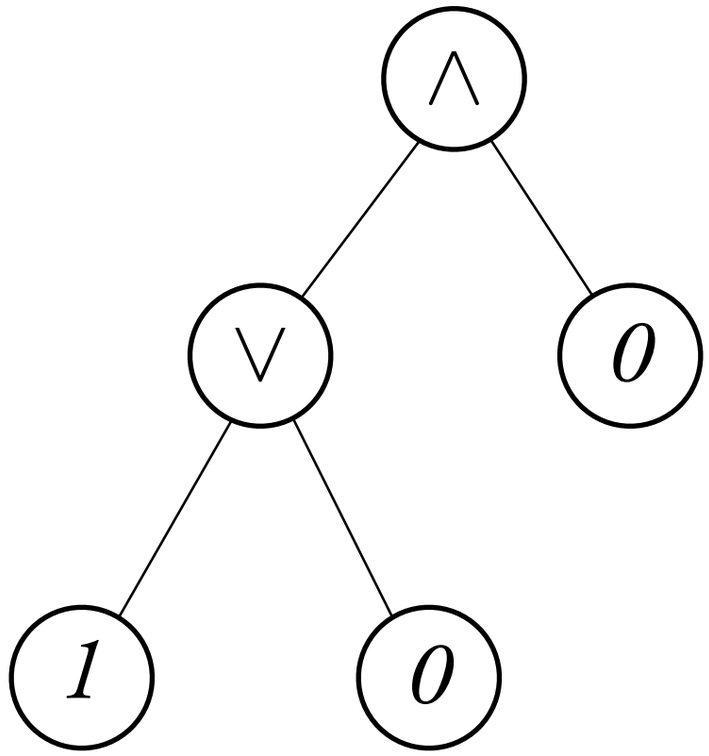

States: $\{q\} \cup (\{left, right\} \times \{0, 1\})$

first time

just evaluated evaluated left/right subtree to 0/1

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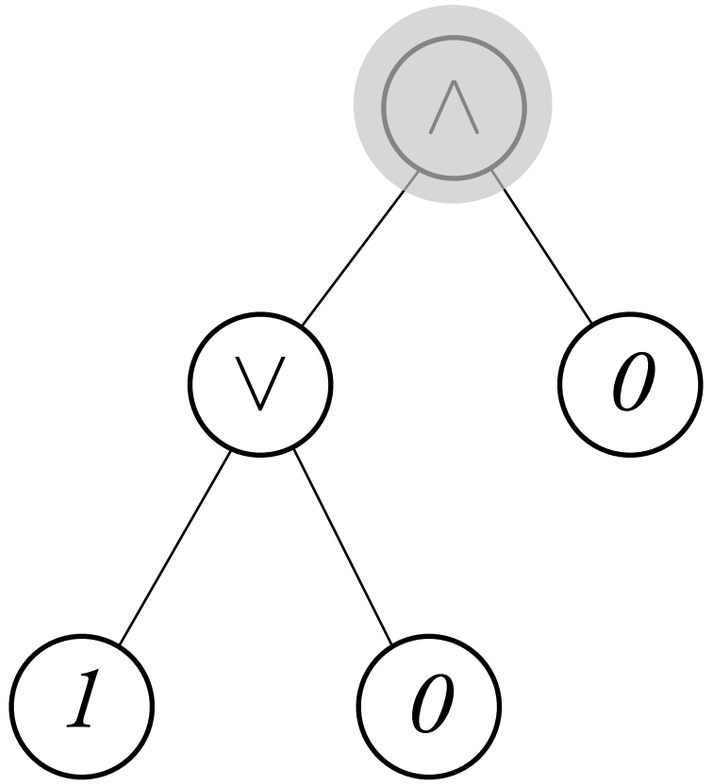

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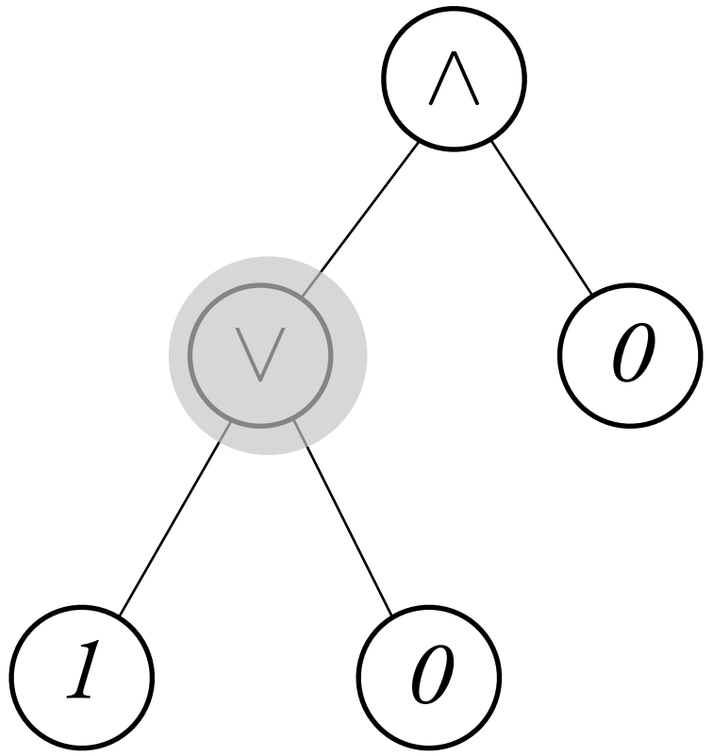

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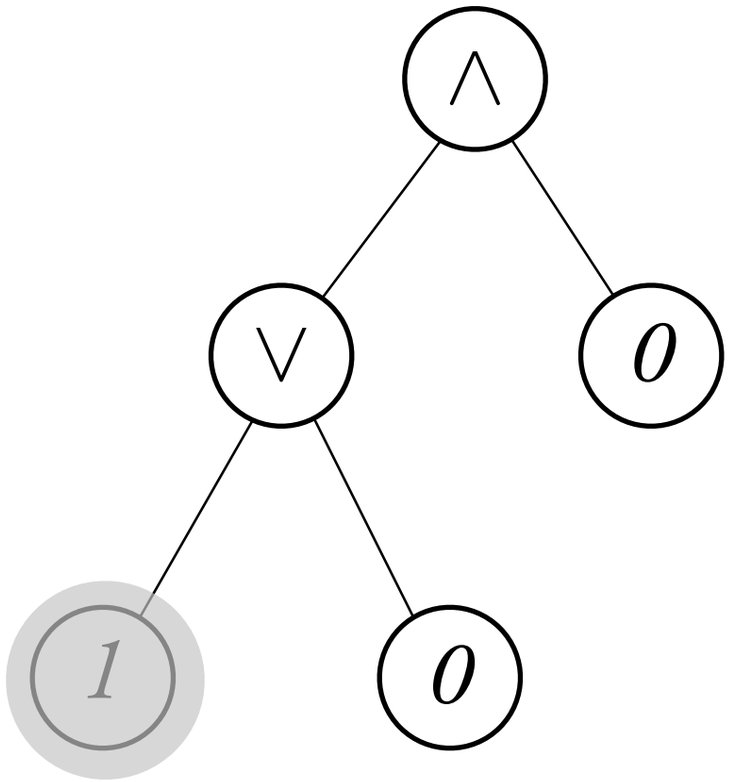

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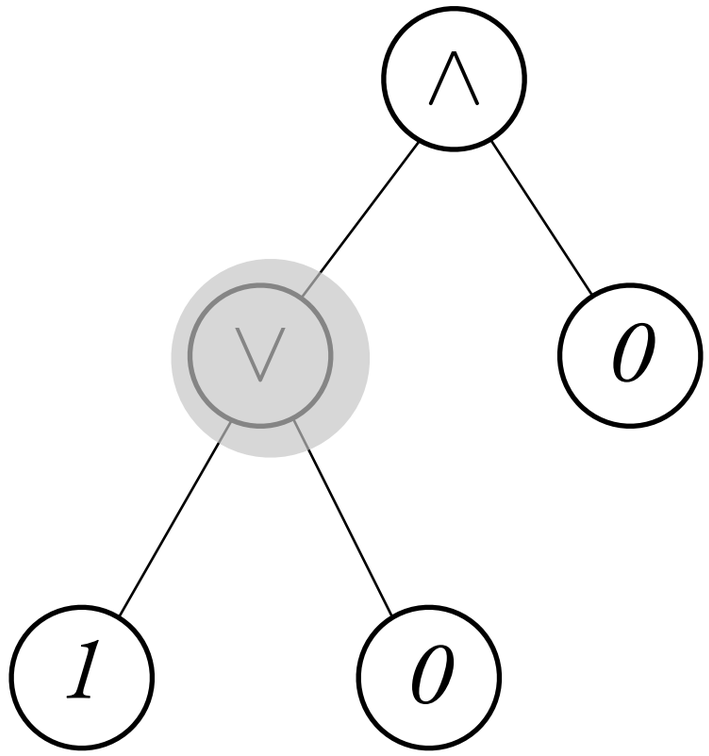

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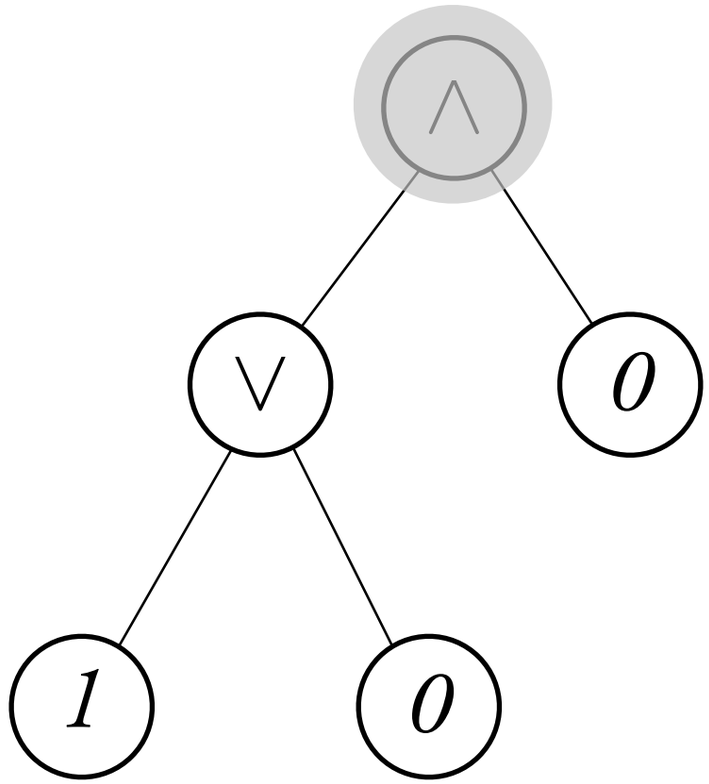

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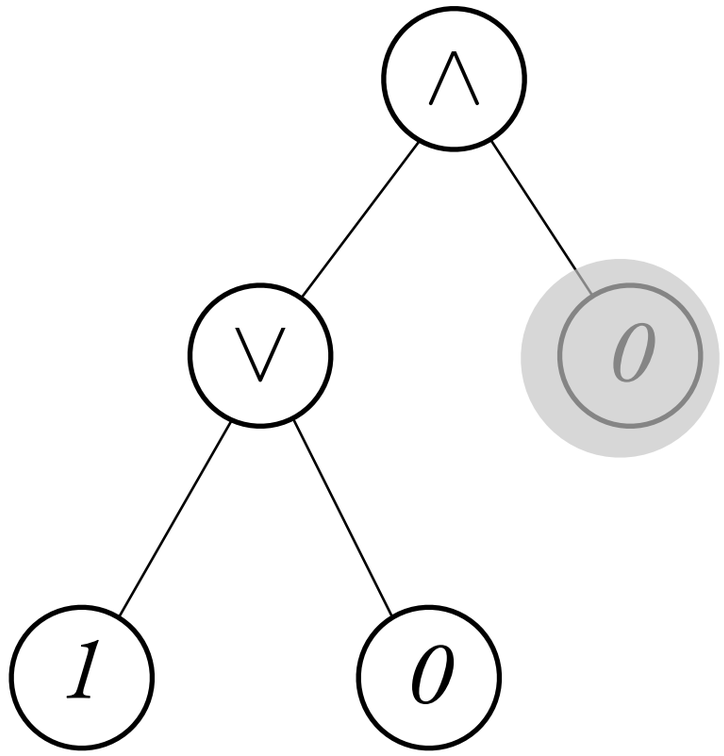

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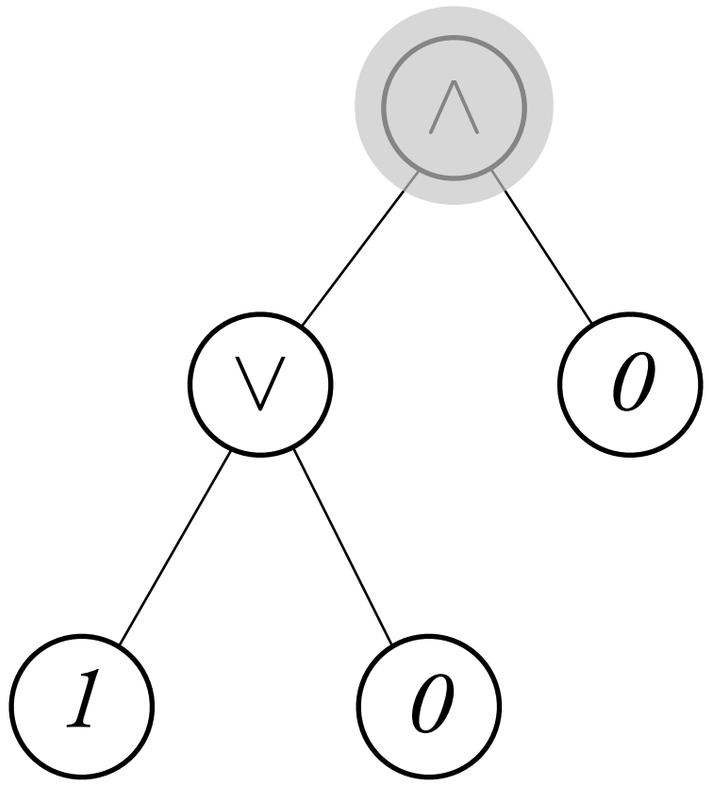

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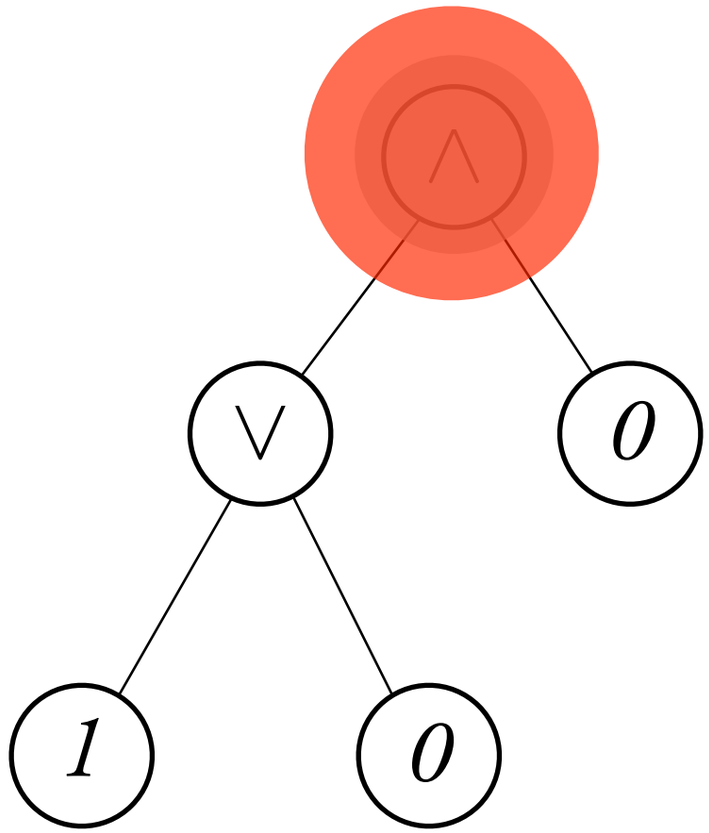

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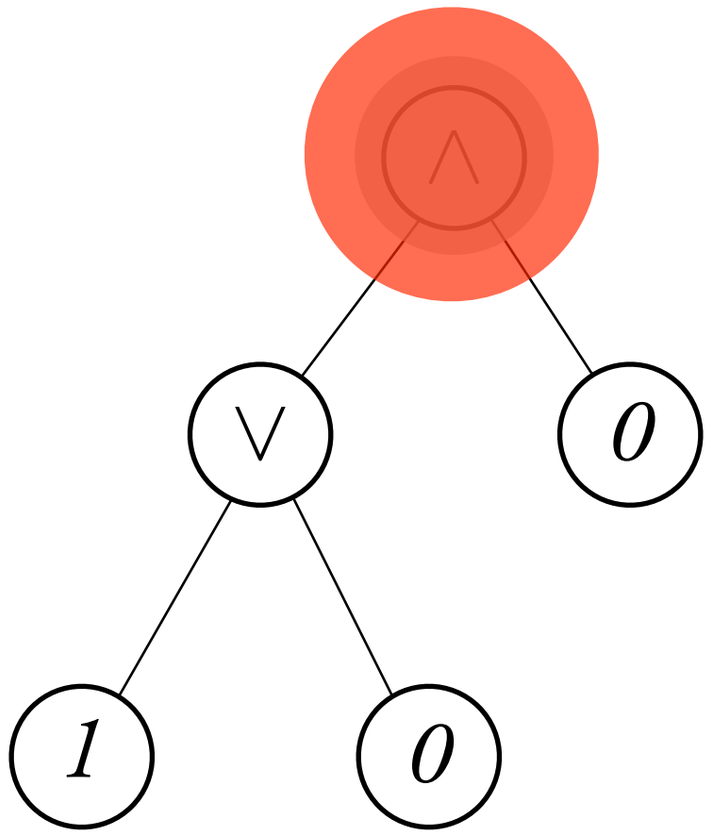

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Open problem:
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Every deterministic tree-walking automaton is equivalent to one that ends every run with a *reject* or *accept* command.

Plan

-A tree-walking automaton

-Expressive power

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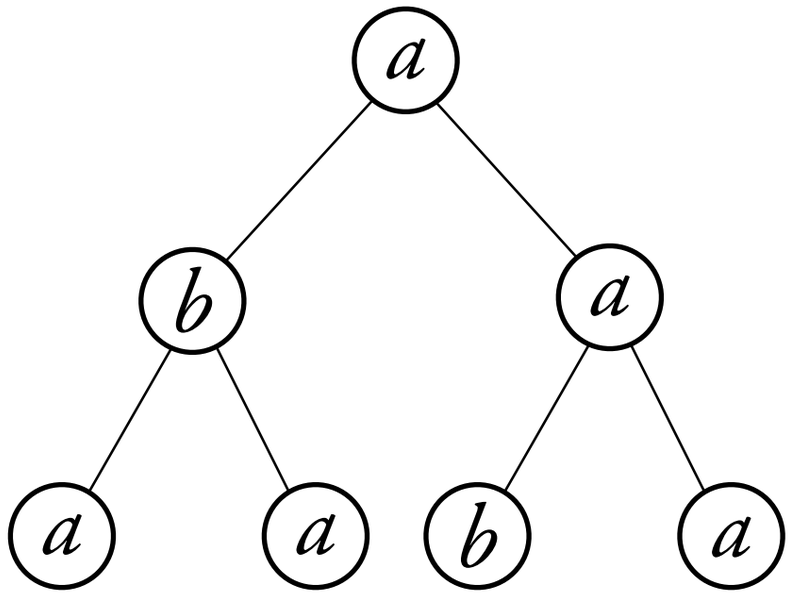
How do tree-walking automata relate to “real” tree automata?



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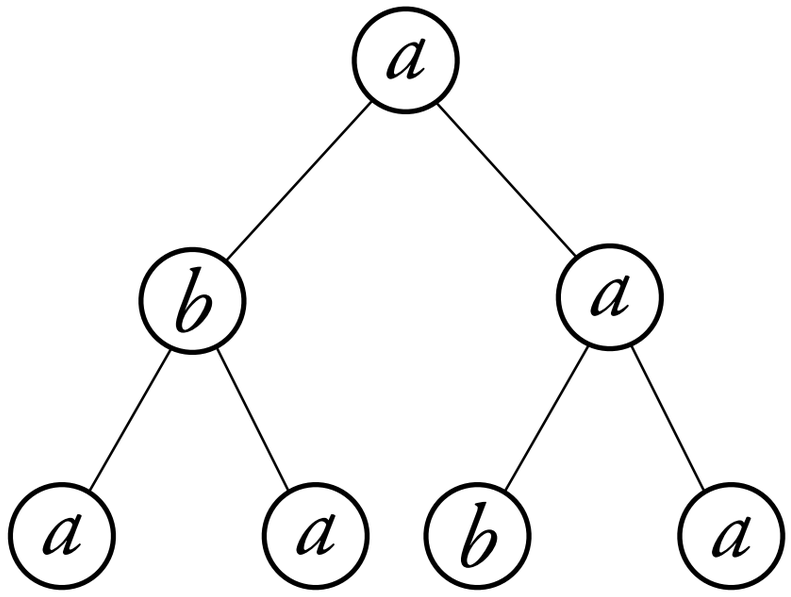

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Here we use bottom-up
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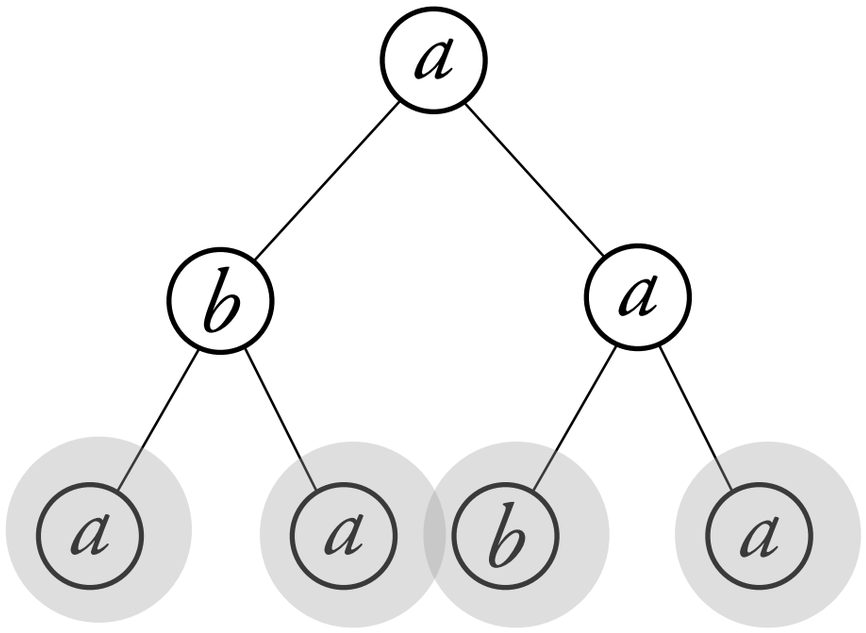

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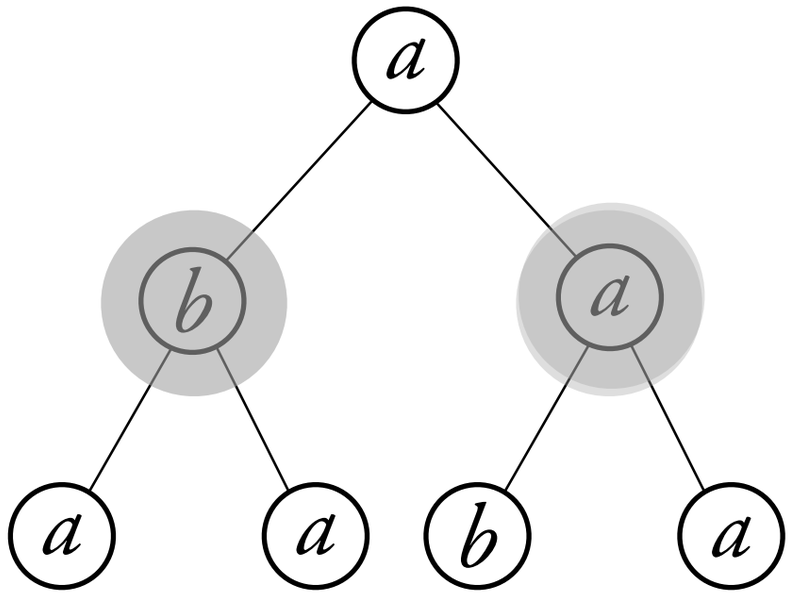

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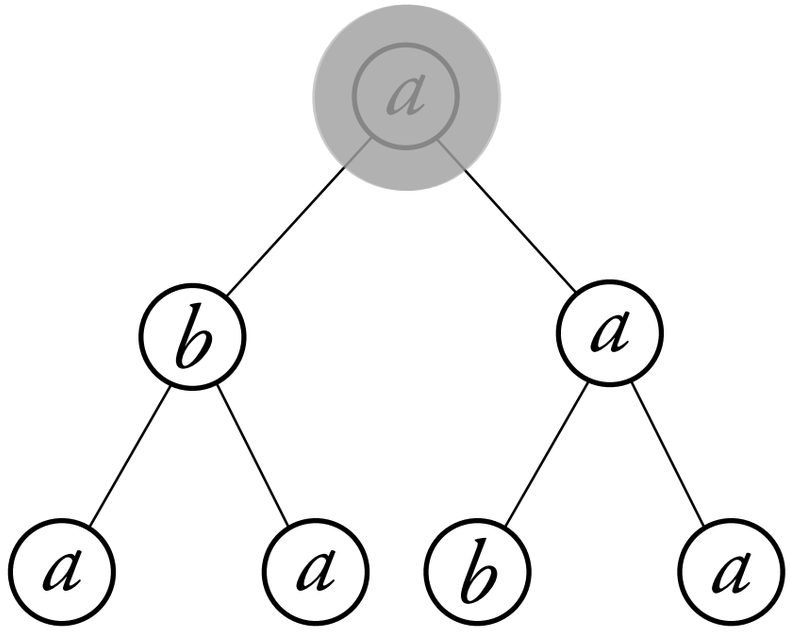

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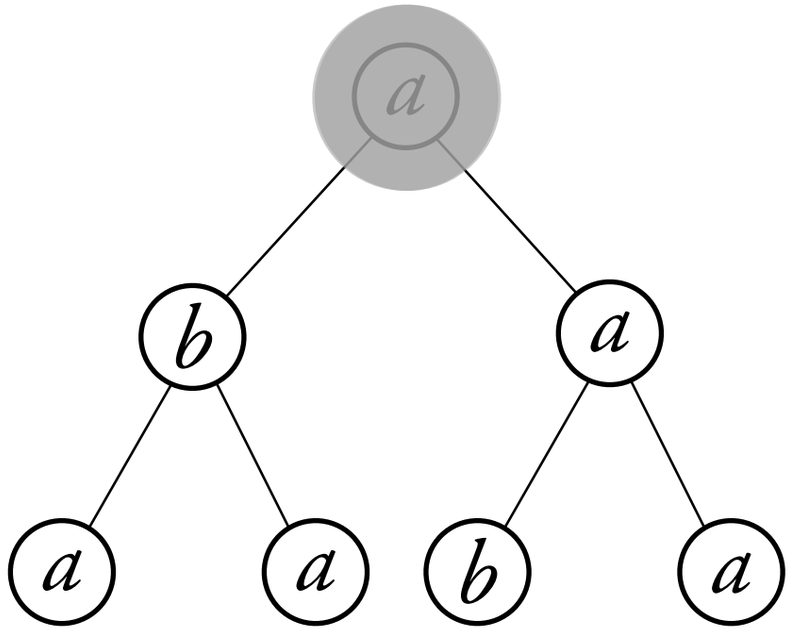

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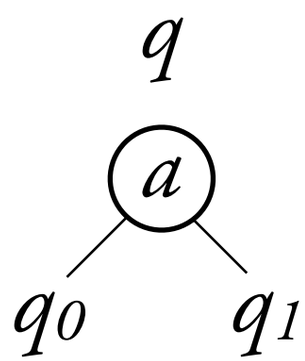

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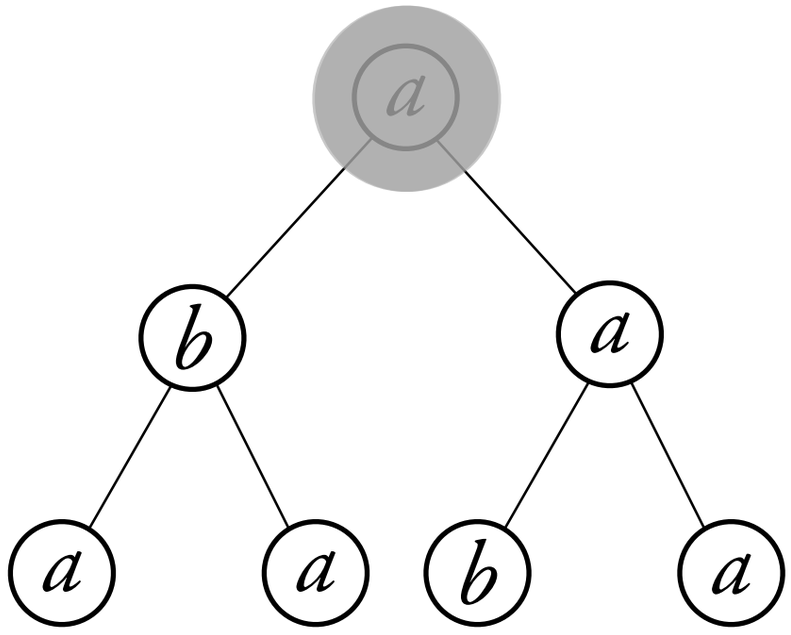

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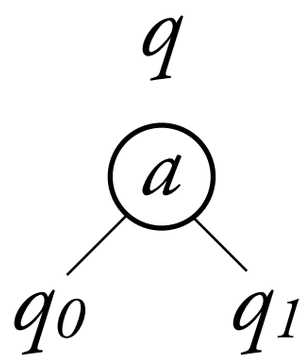
$$\langle Q, q_I, \Sigma, \delta, F \rangle$$



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Branching automata are closed under union, intersection, complementation, projection etc.

Def. A tree language is called *regular* if it is recognized by a branching automaton.

Question: how do tree-walking automata relate to regular languages?

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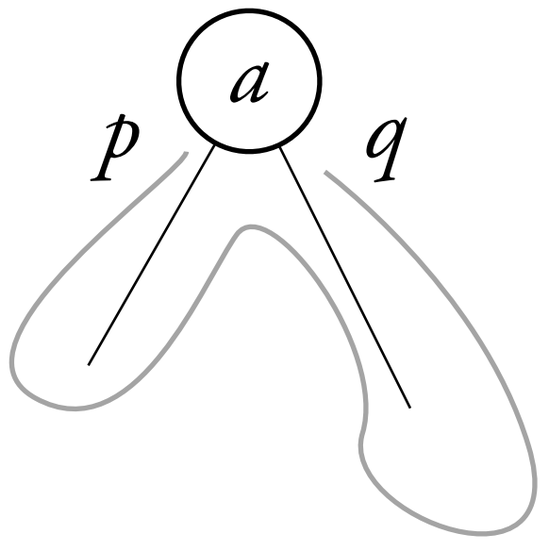

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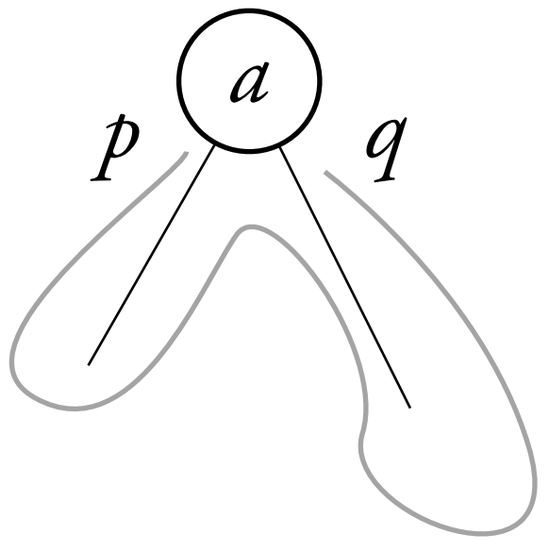

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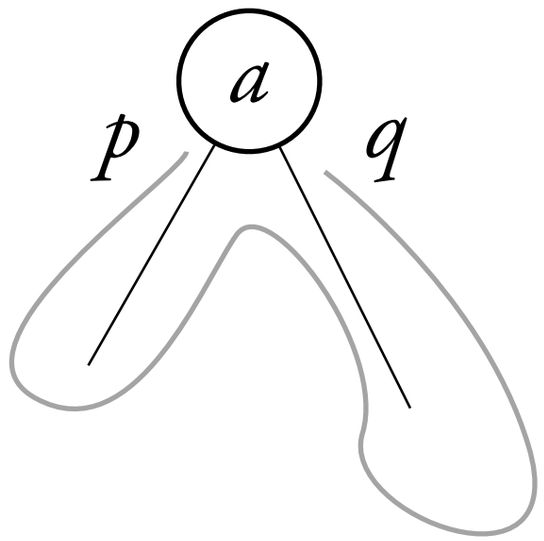

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Corollary. Emptiness for tree-walking automata is in EXPTIME.

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Hardness. Reduction from APSPACE.

For an alternating Turing machine that uses n memory cells, we write a tree-walking automaton with equivalent emptiness and $O(n)$ states.

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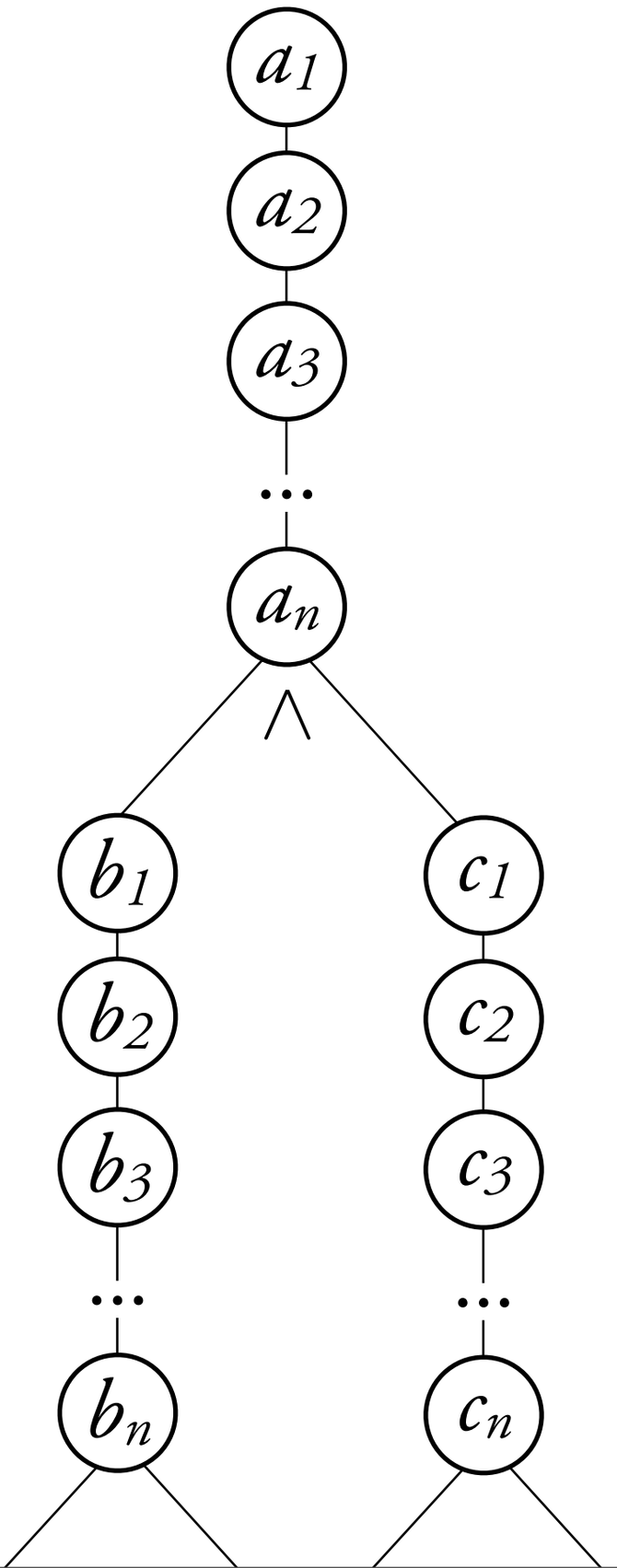

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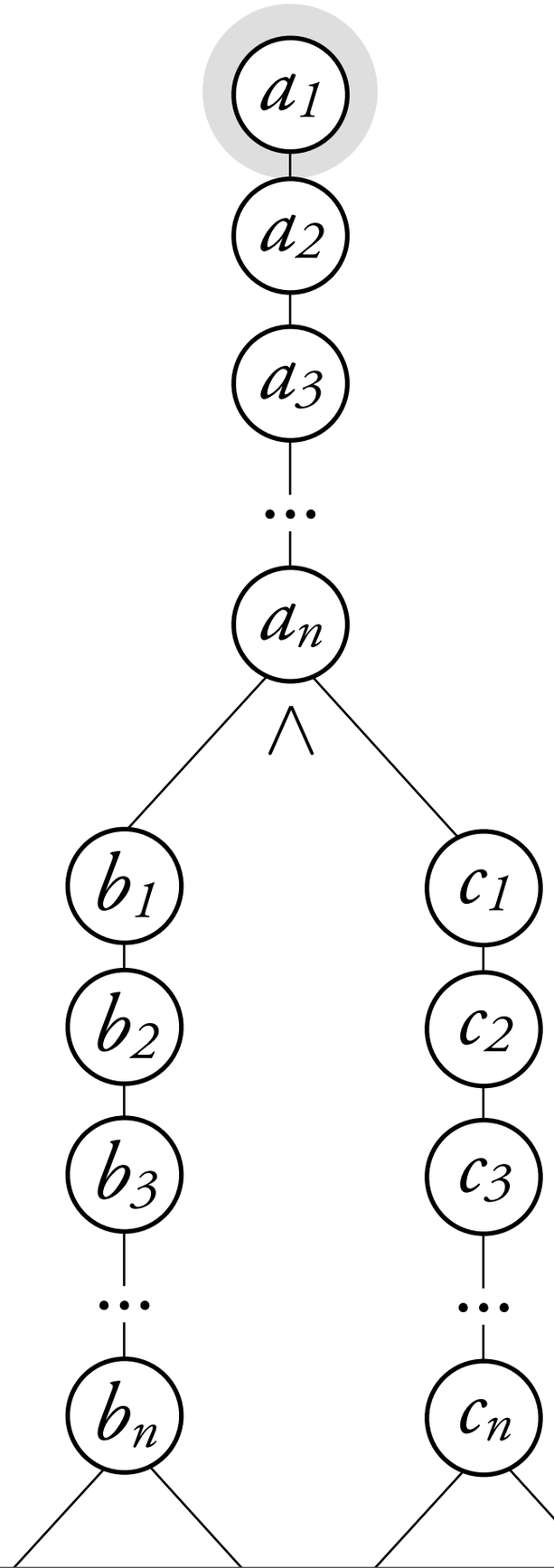

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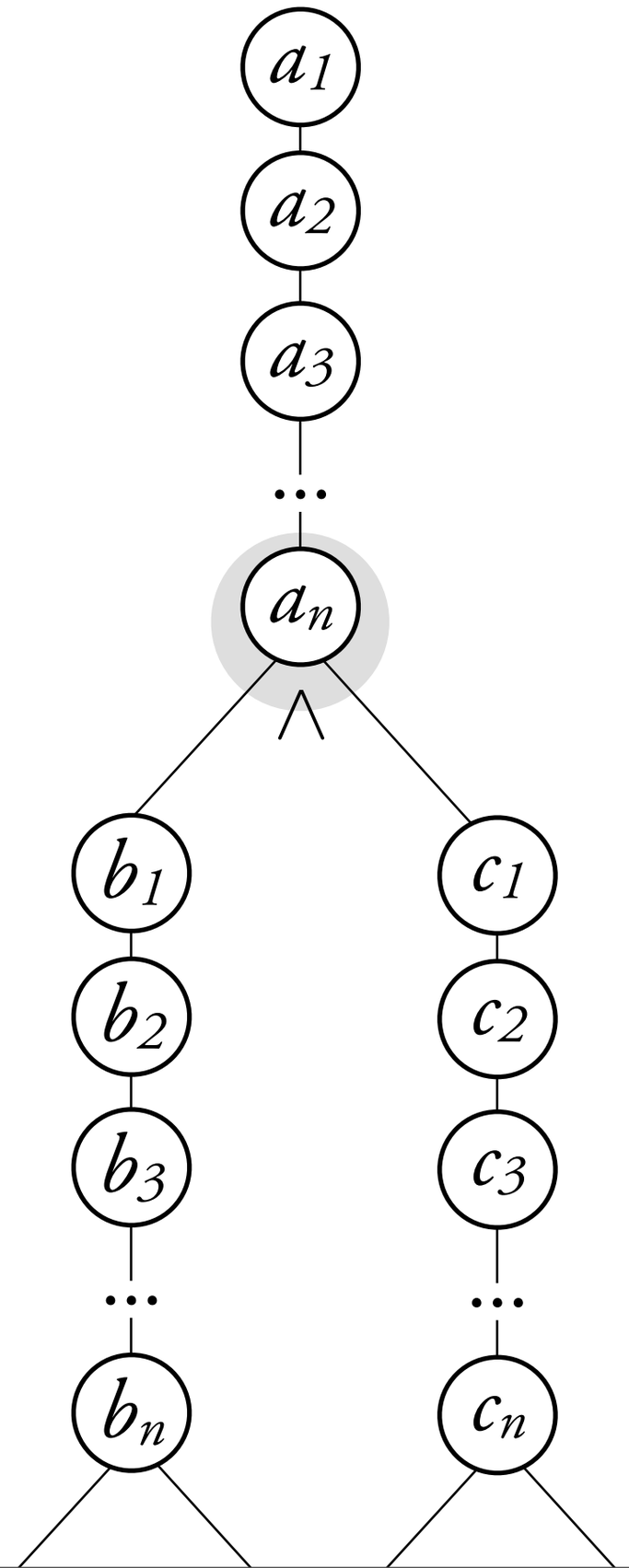

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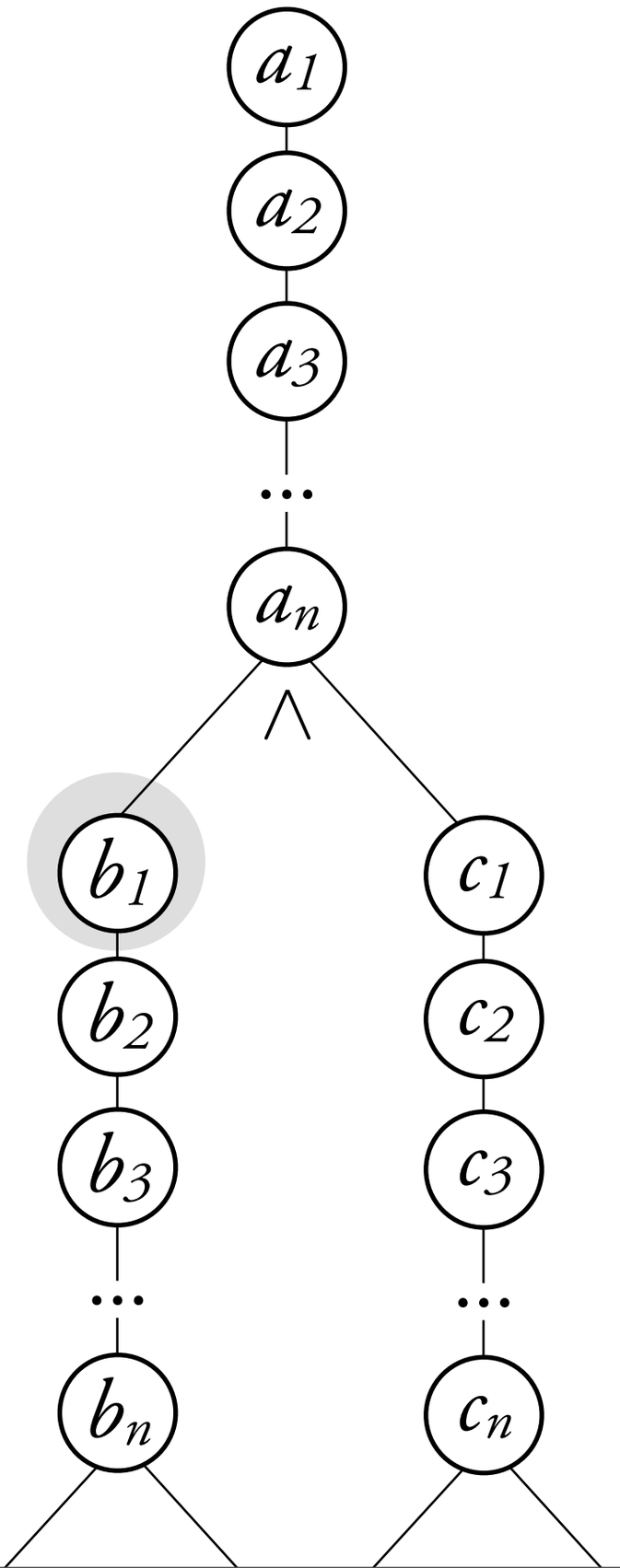

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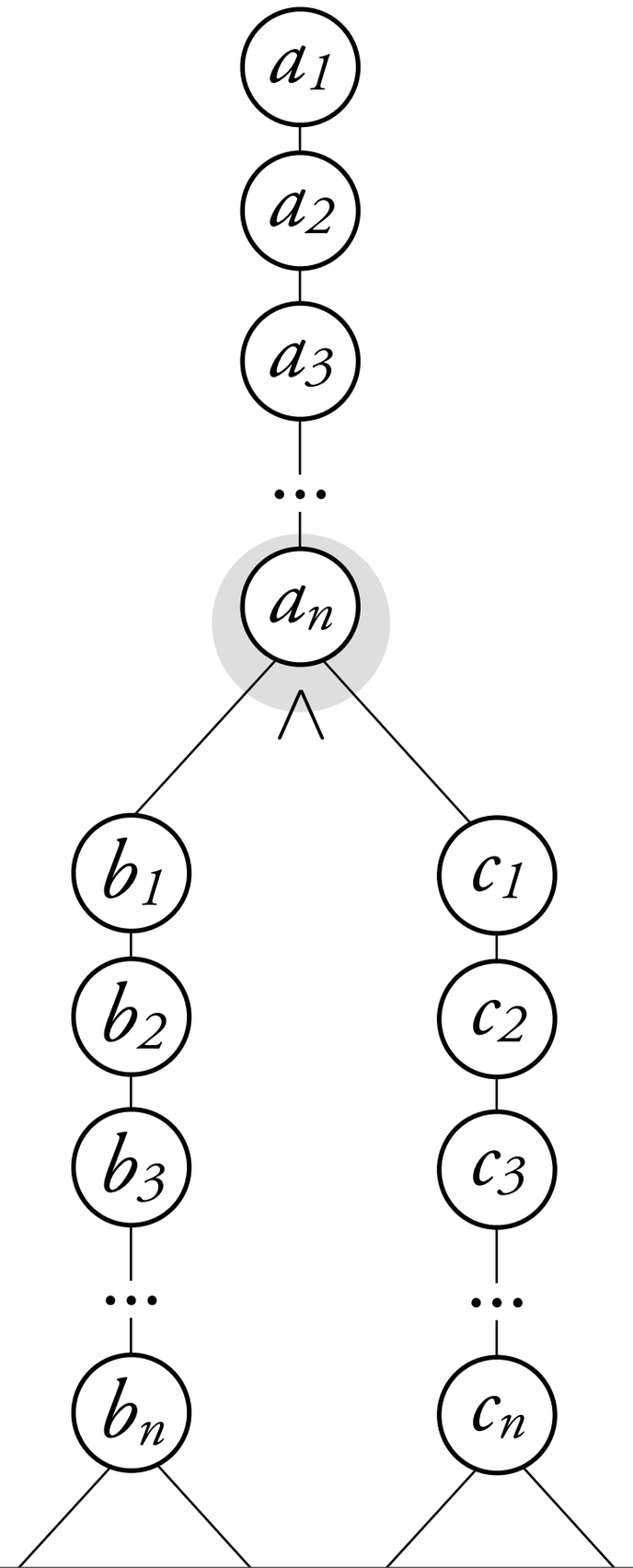

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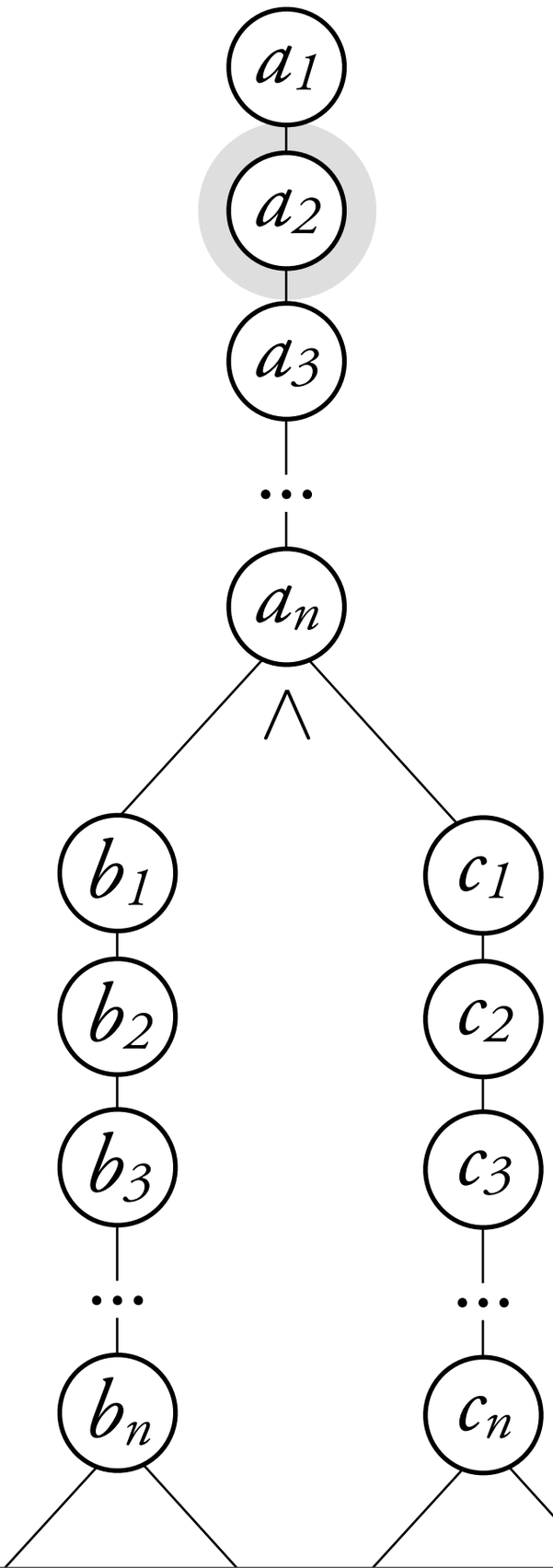

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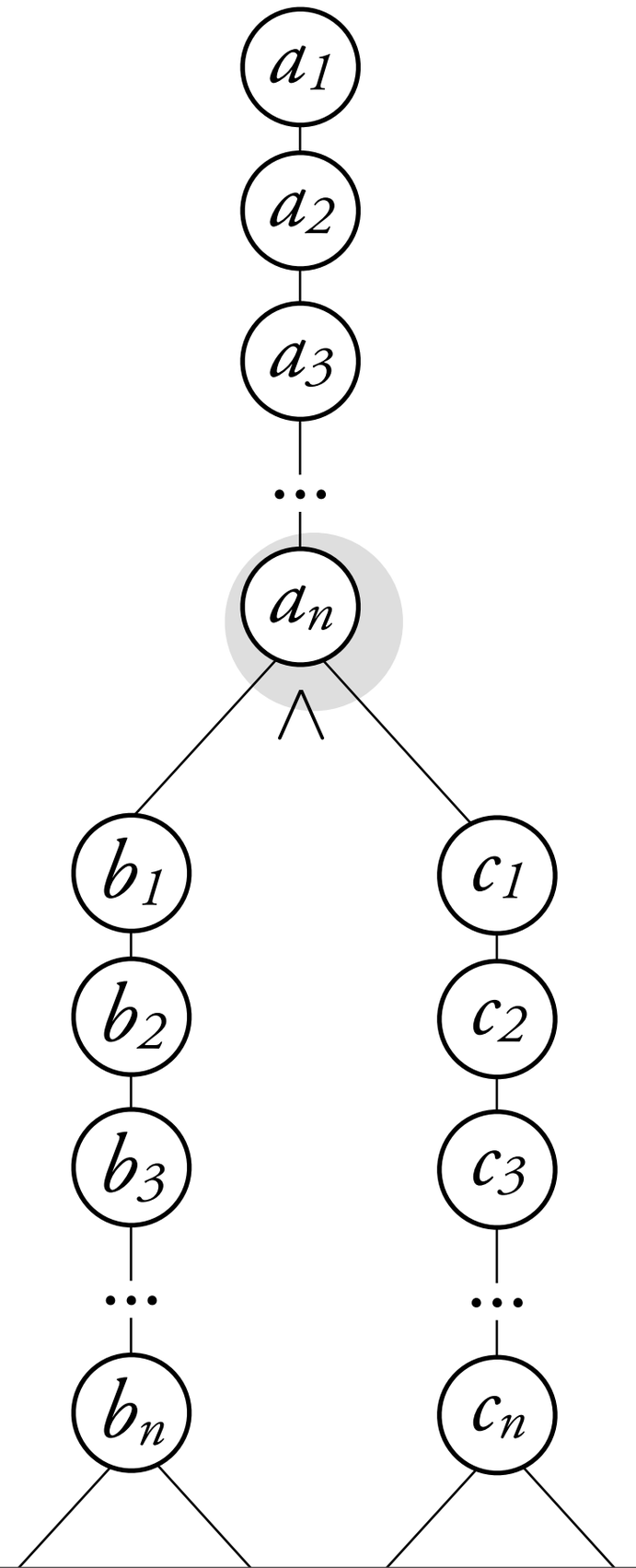

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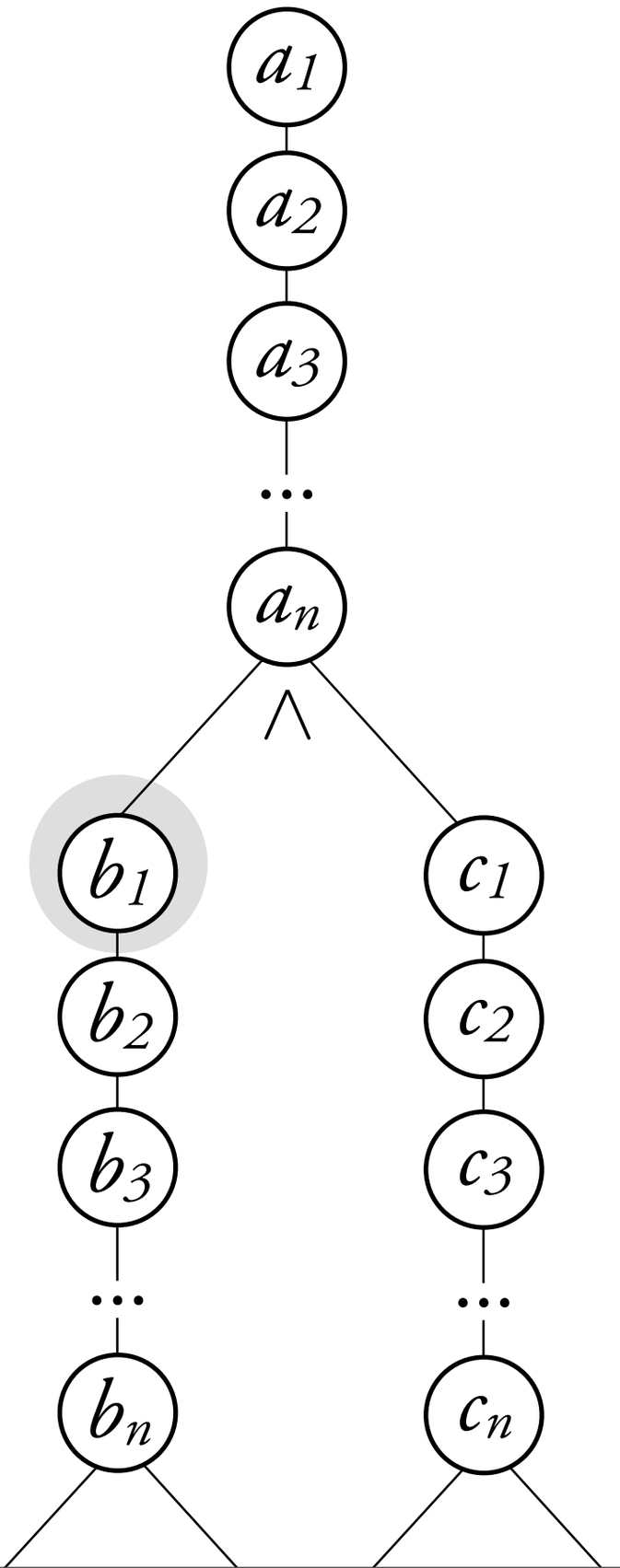

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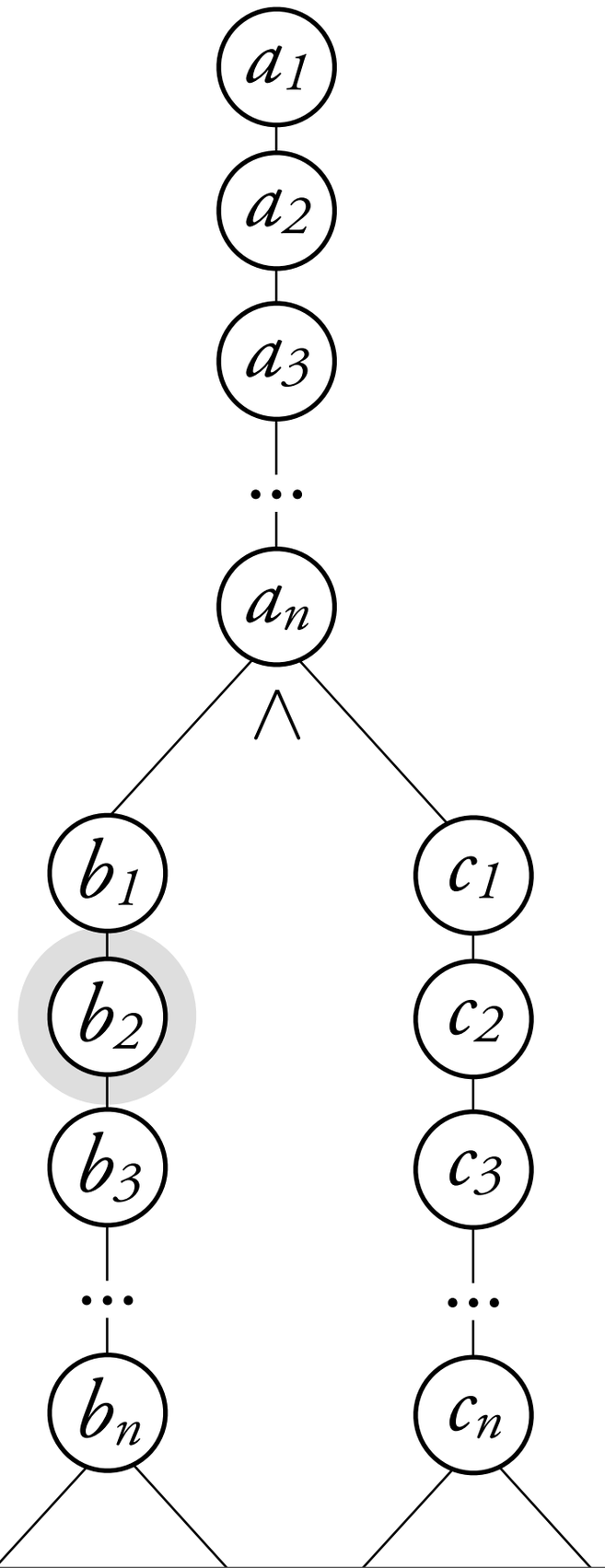

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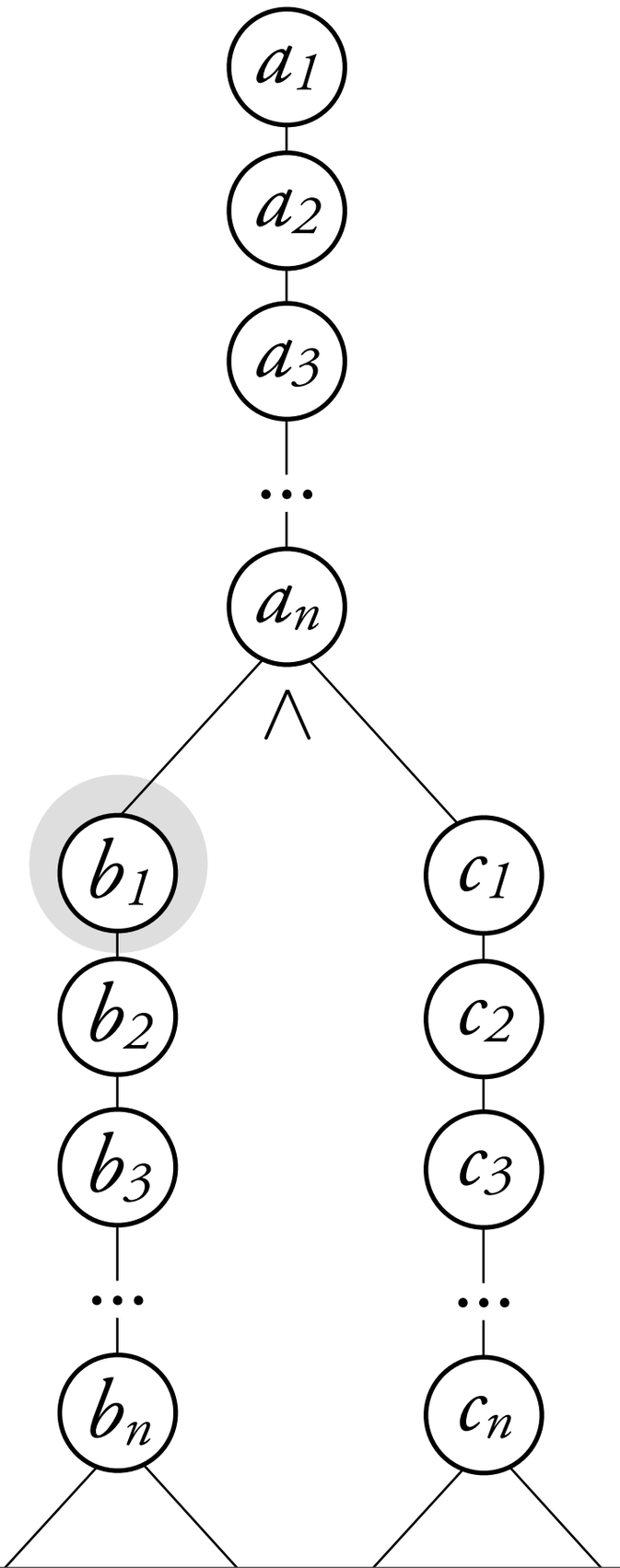

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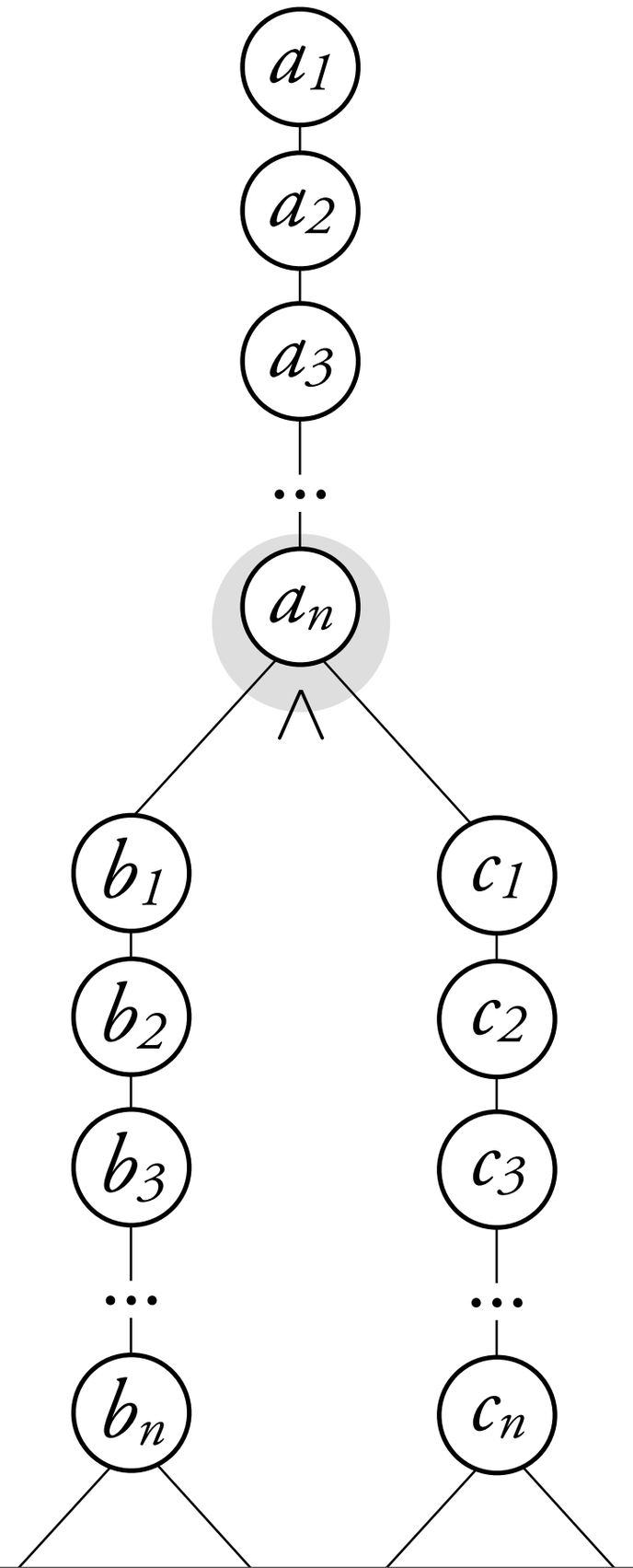

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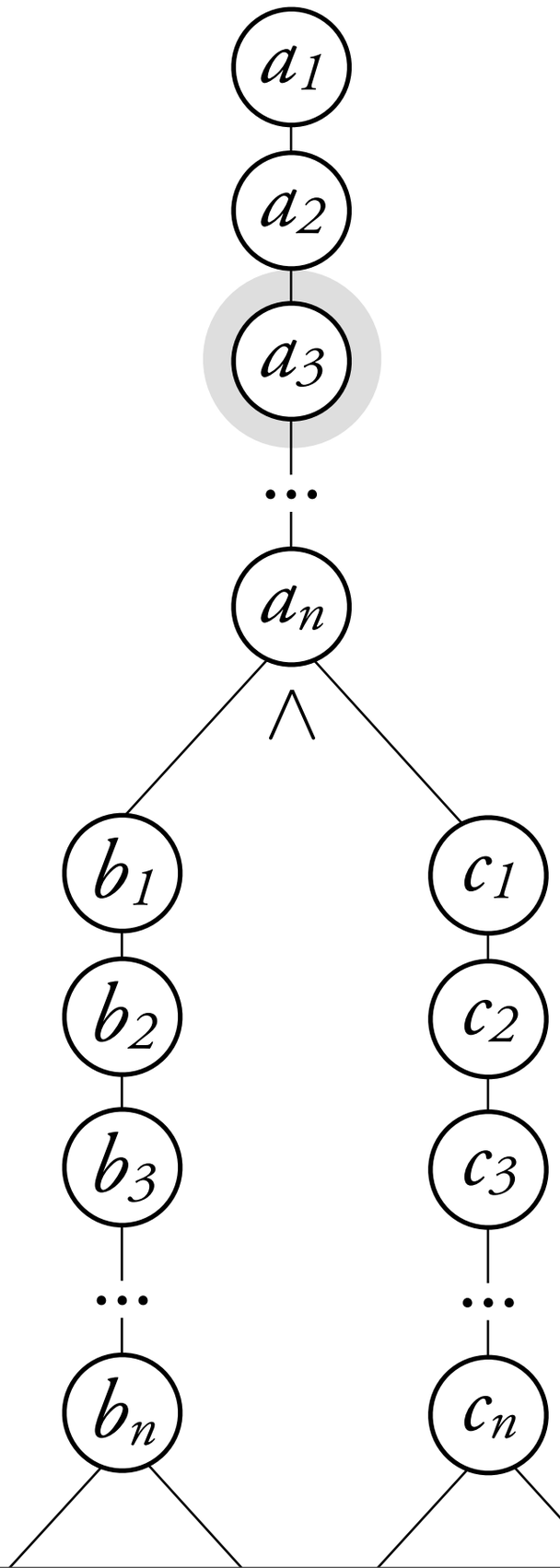

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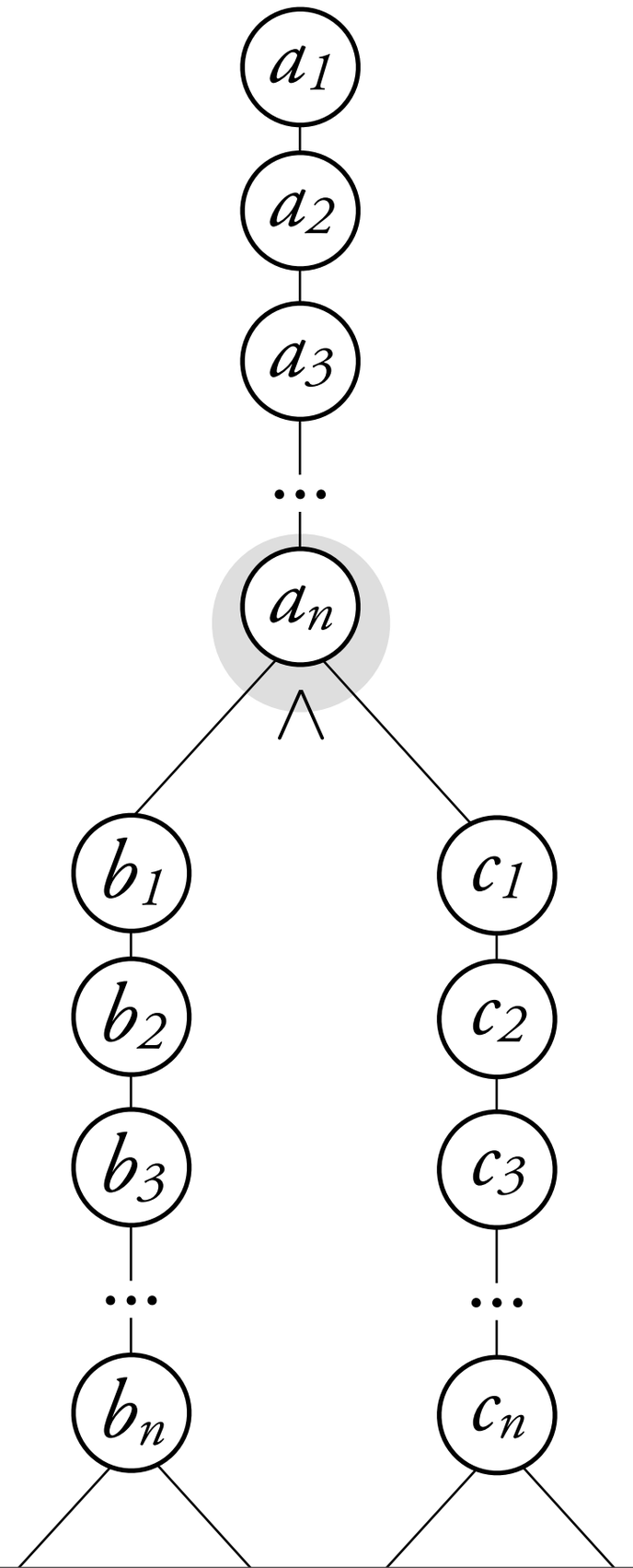

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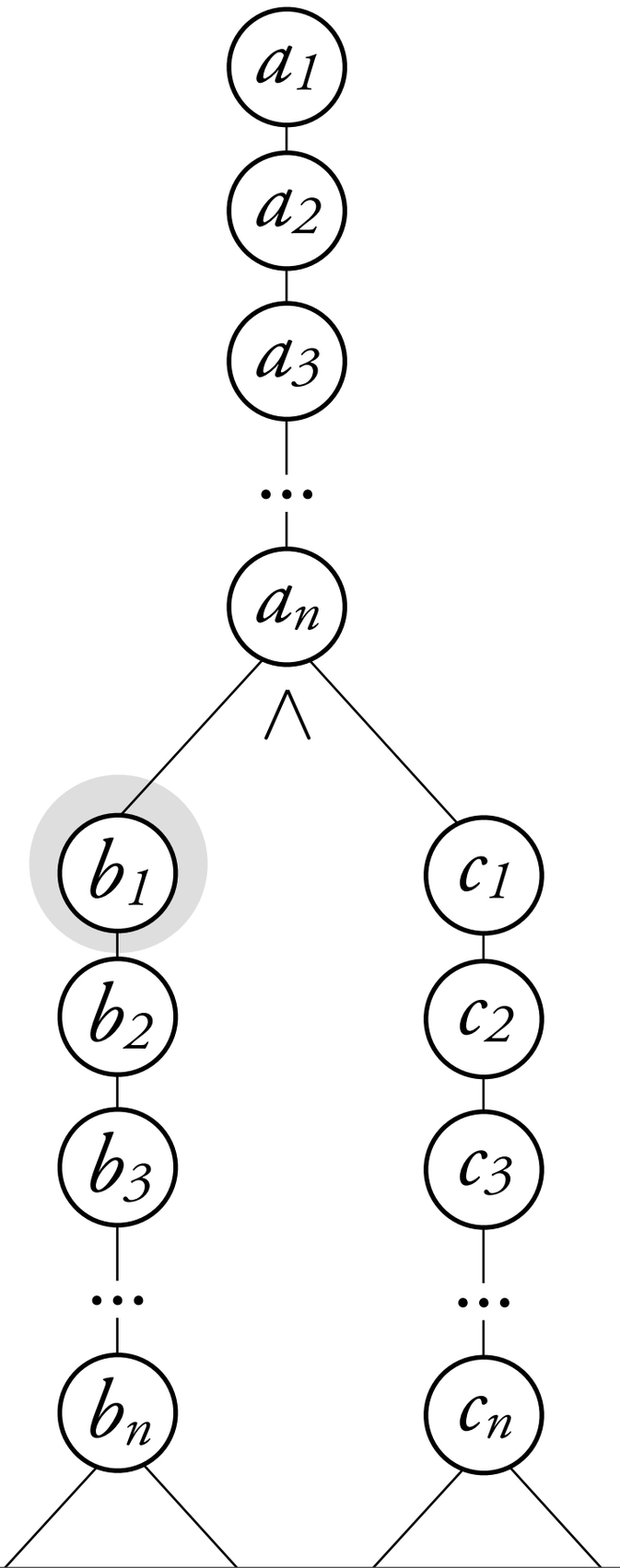

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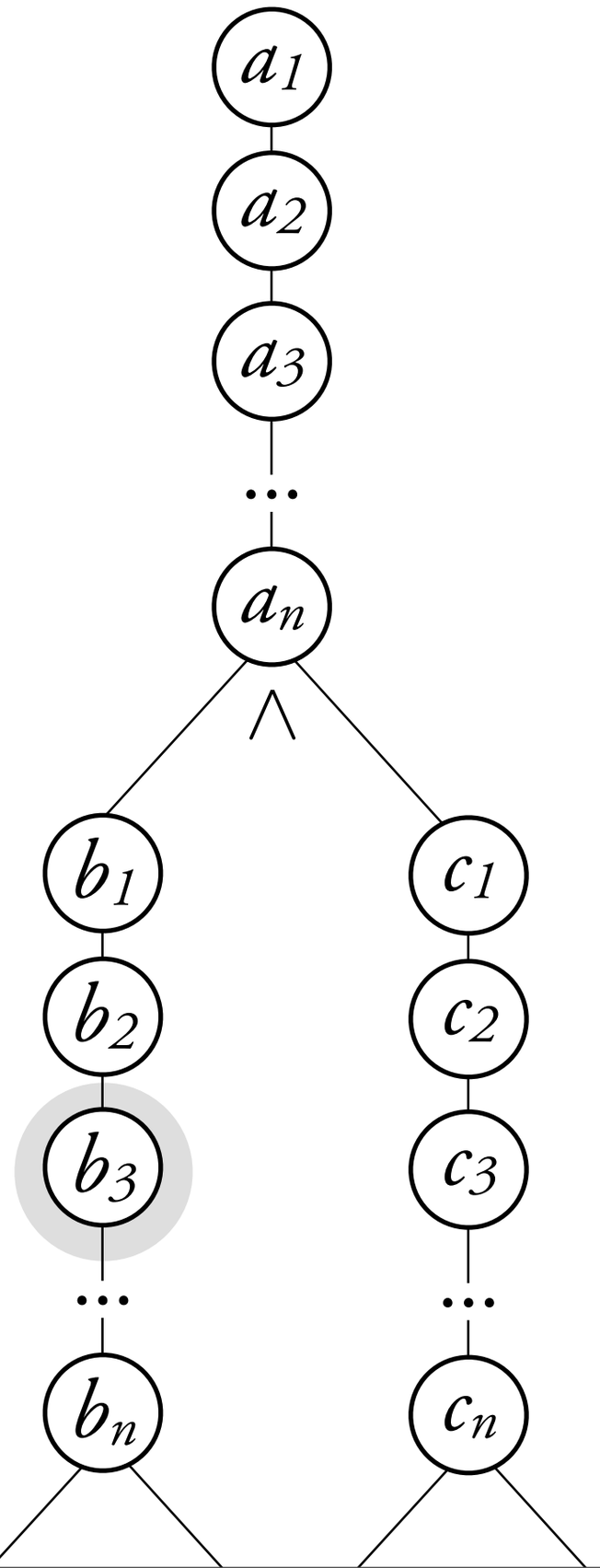

TWA $\subseteq$ REG

Is the inclusion strict?

Theorem (B., Colcombet '05) The inclusion is strict.

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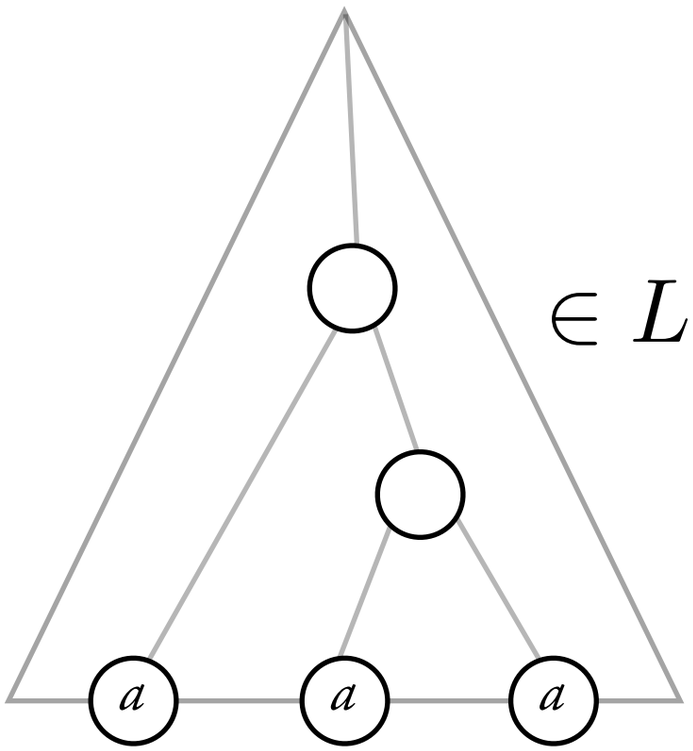

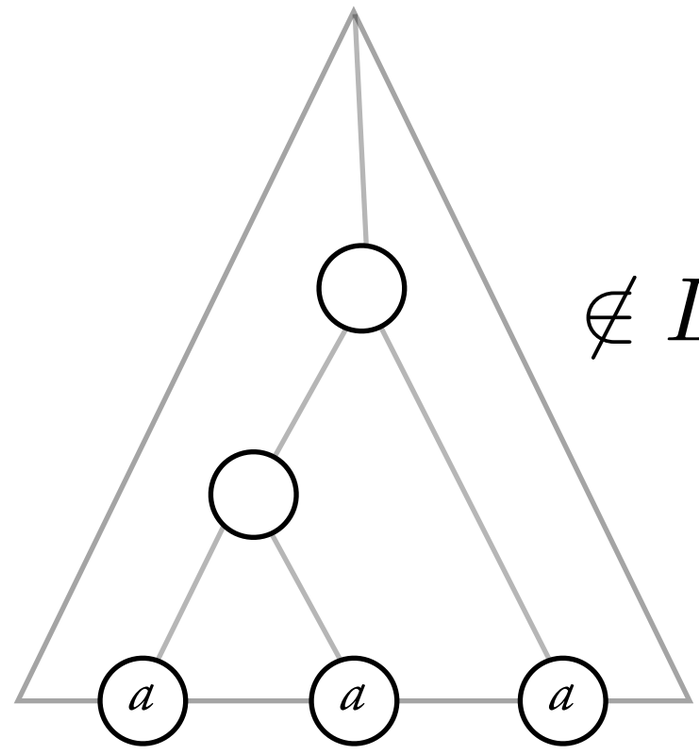

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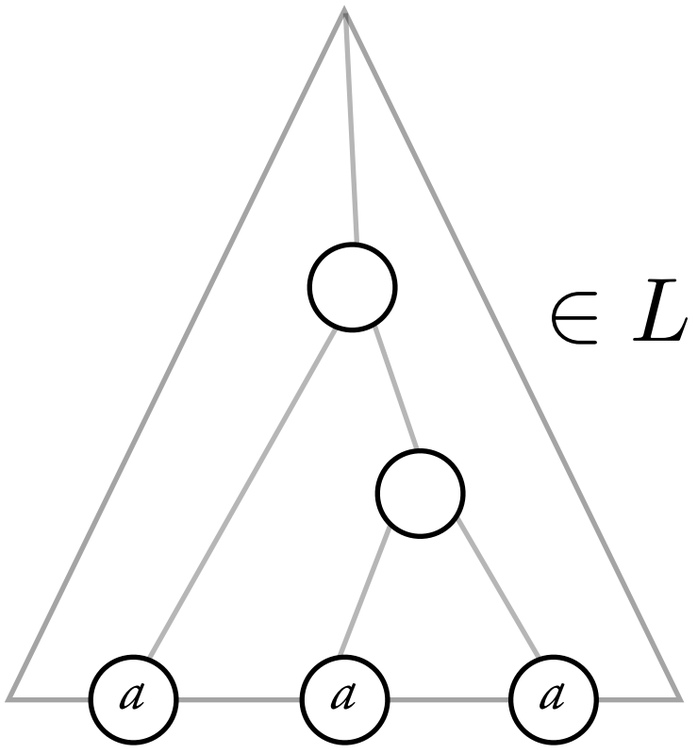

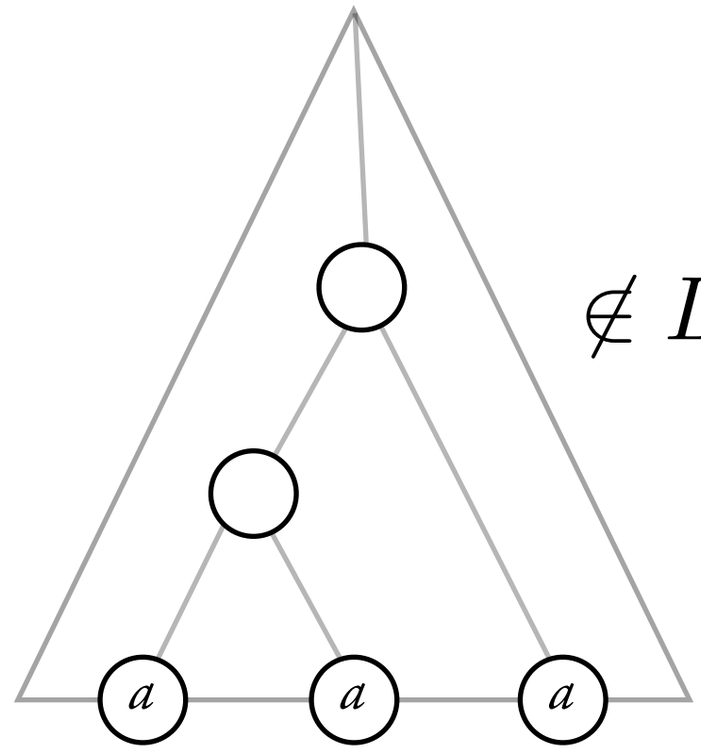

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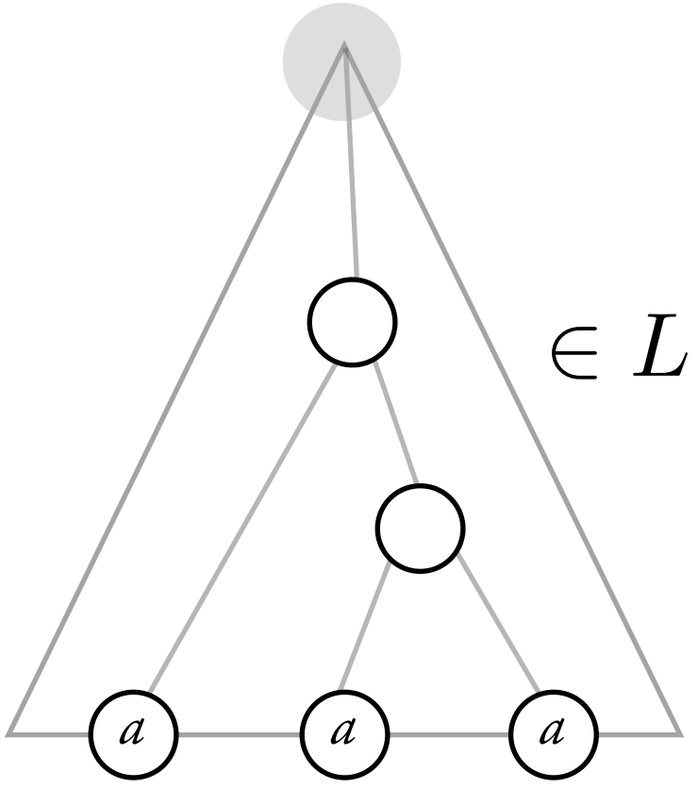

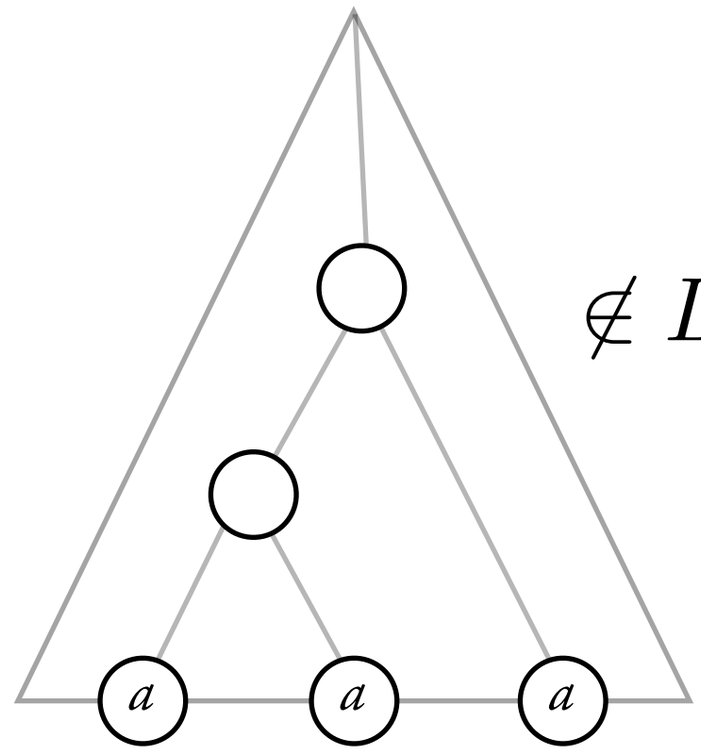

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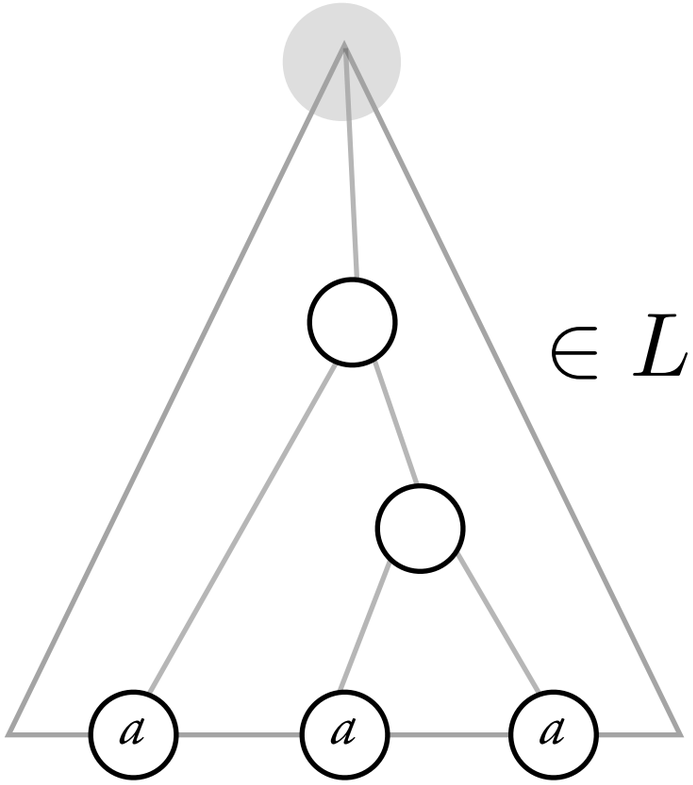

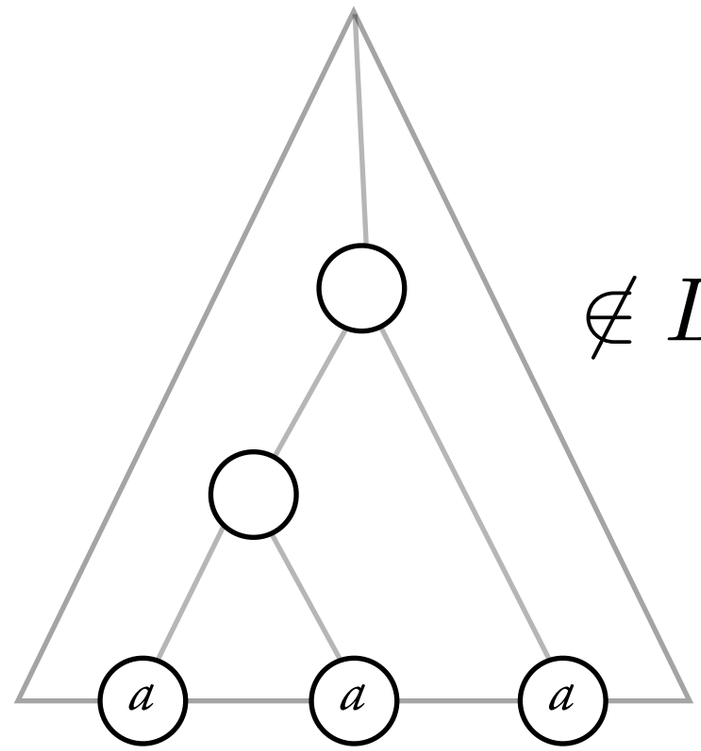

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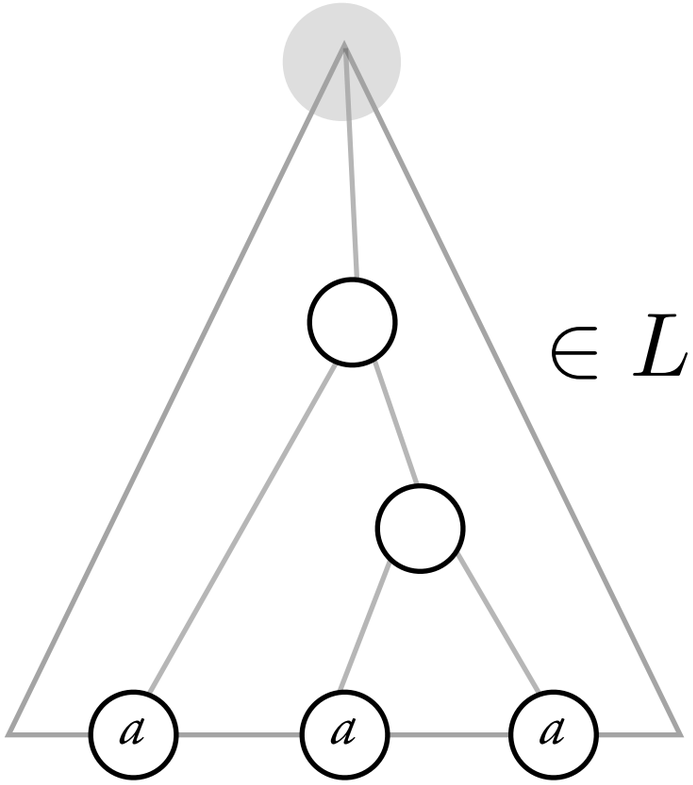

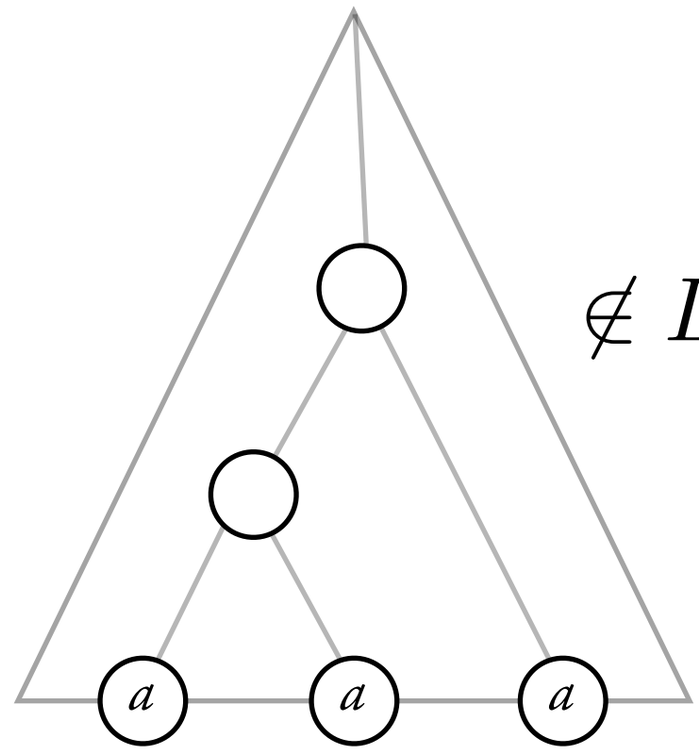

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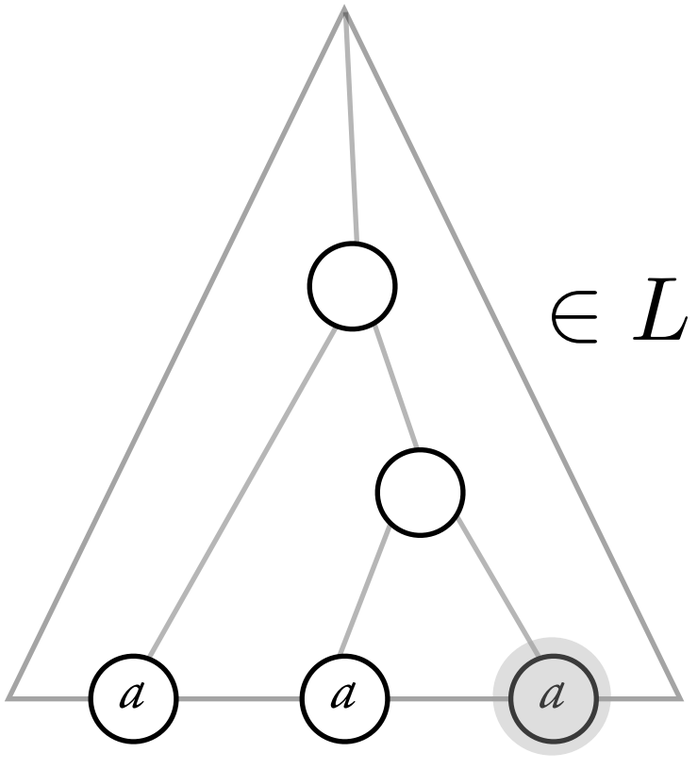

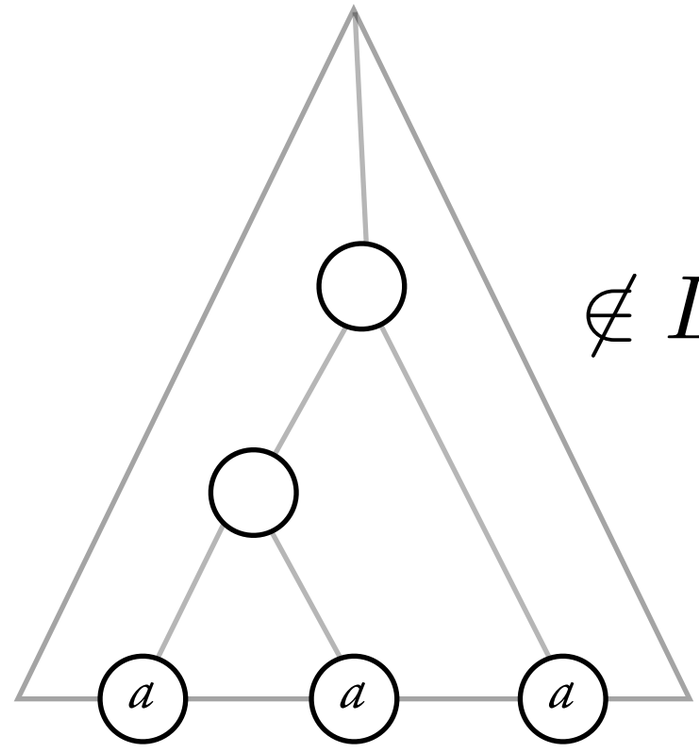

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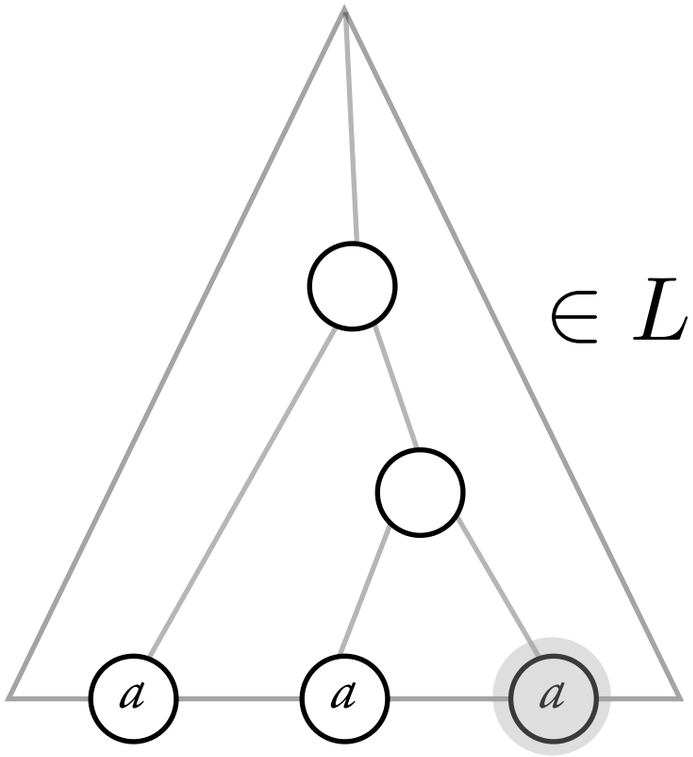

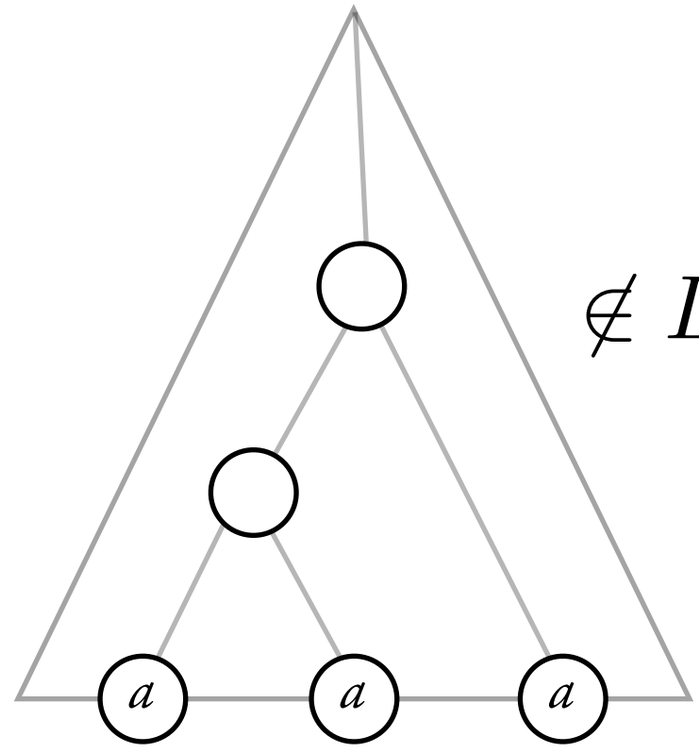

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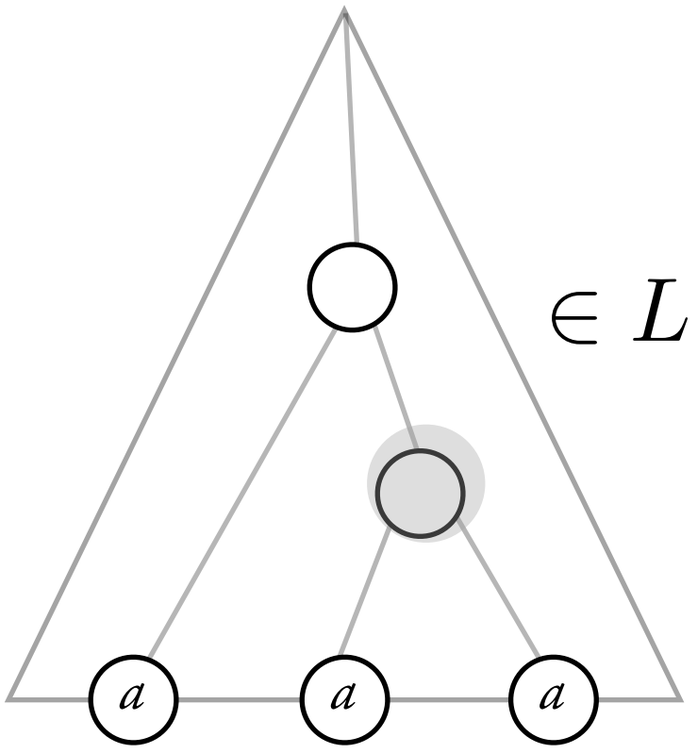

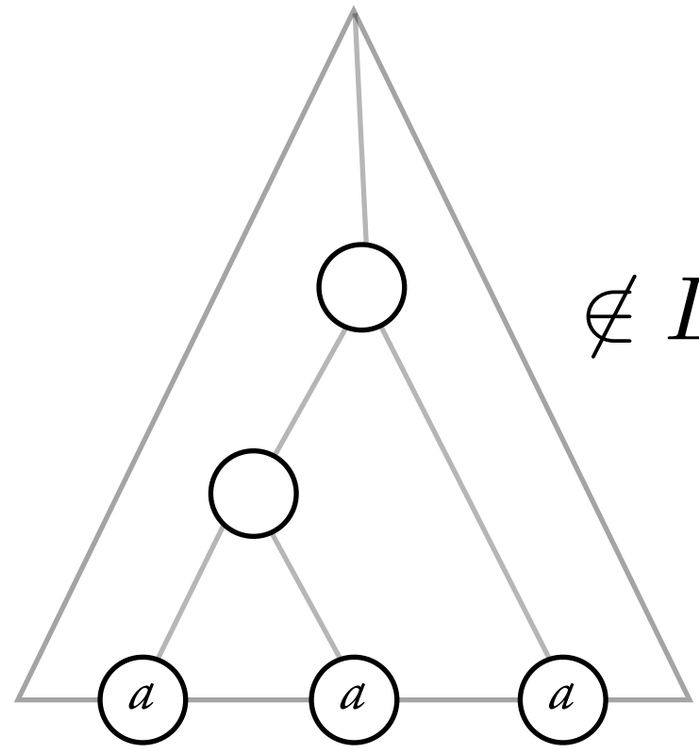

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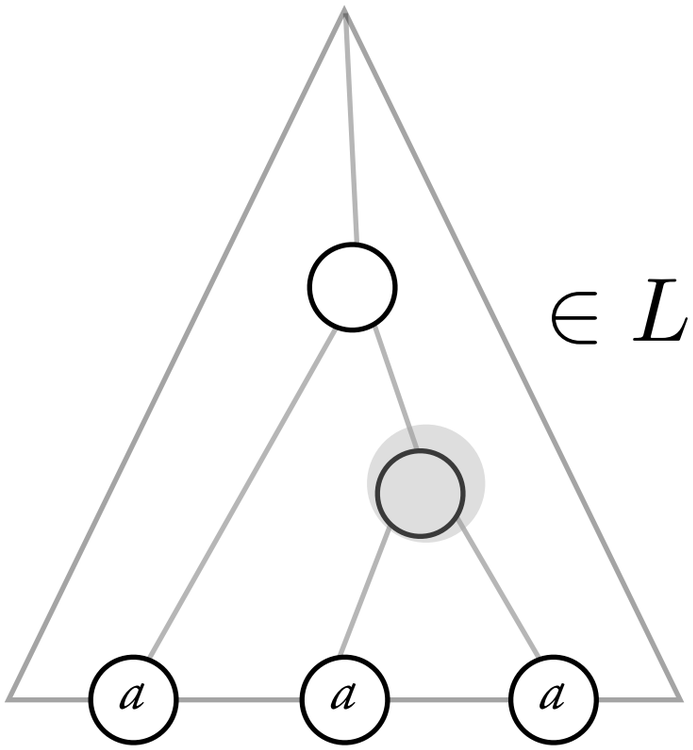

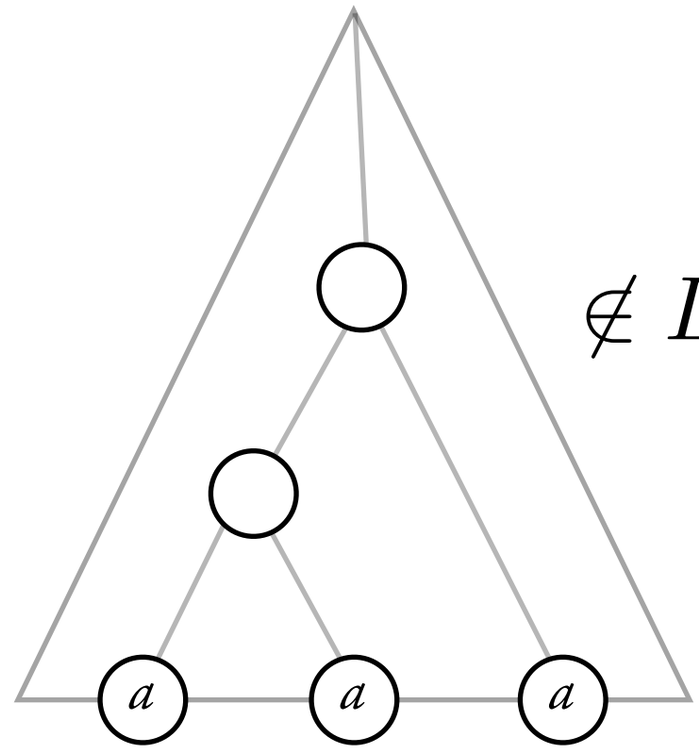

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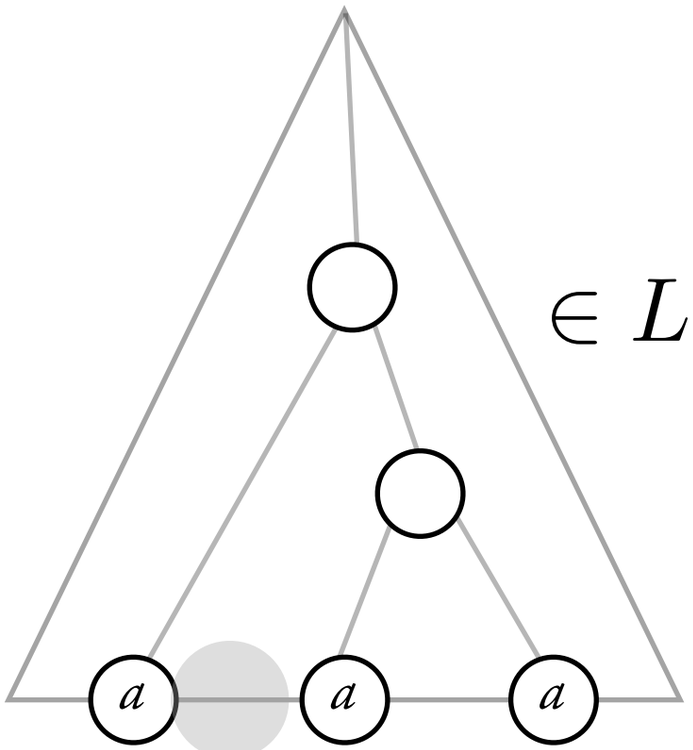

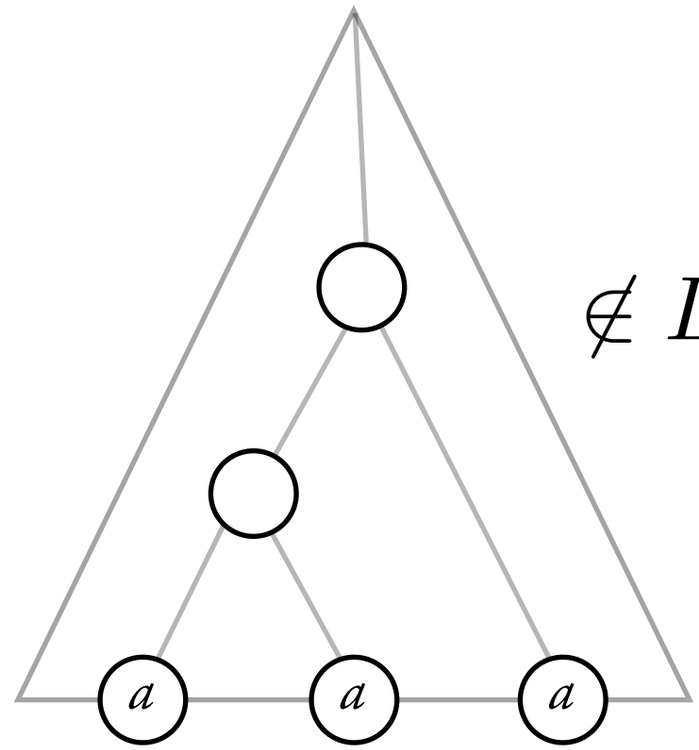

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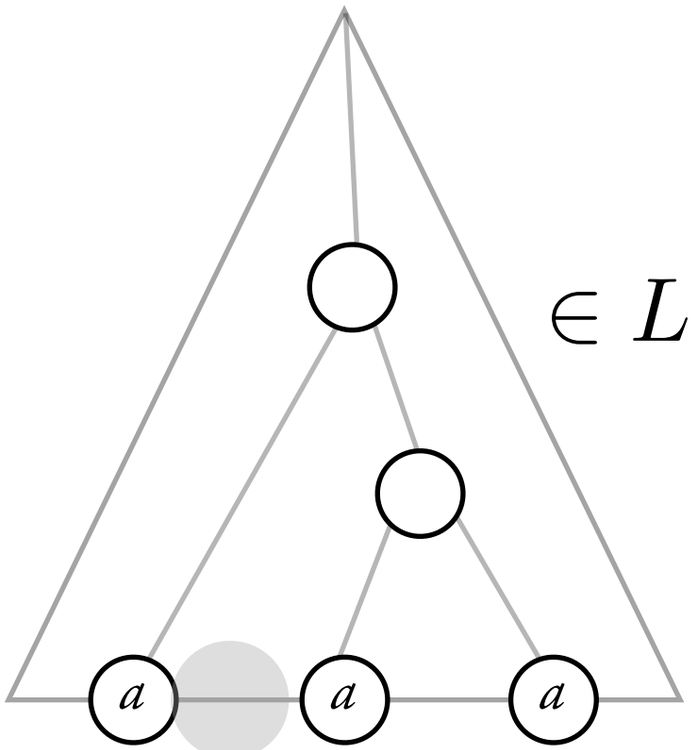

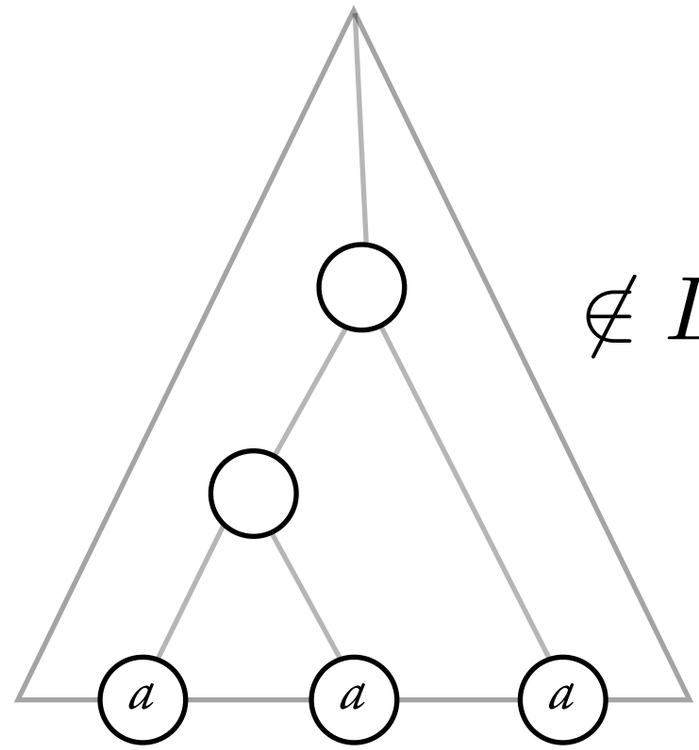

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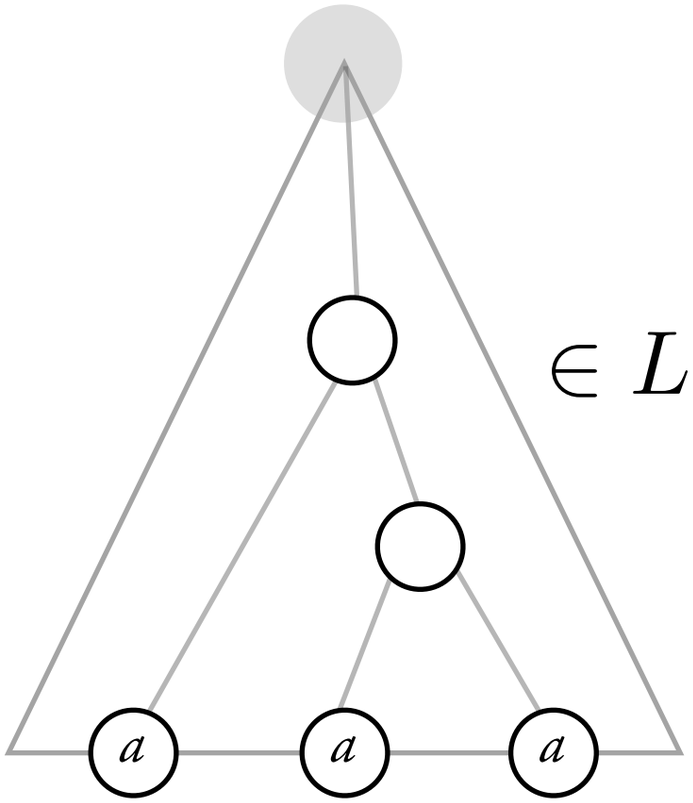

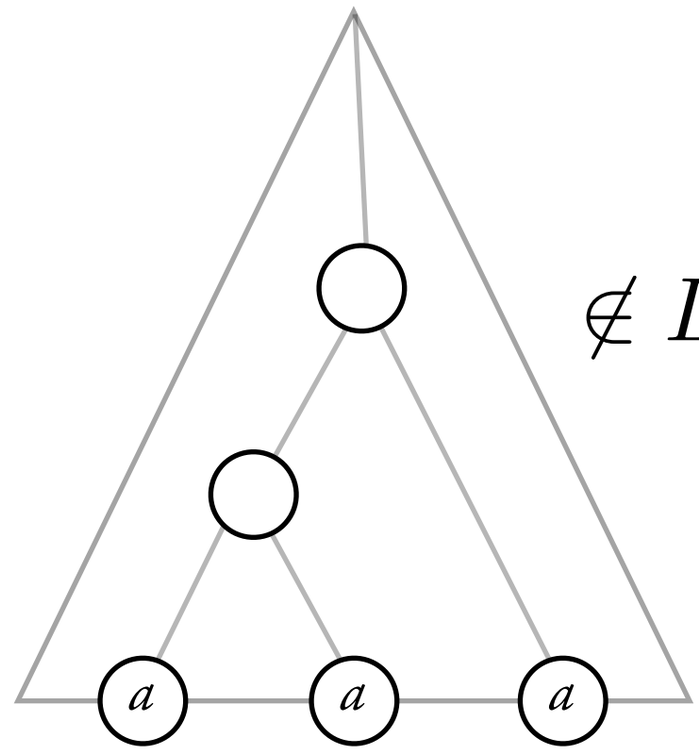

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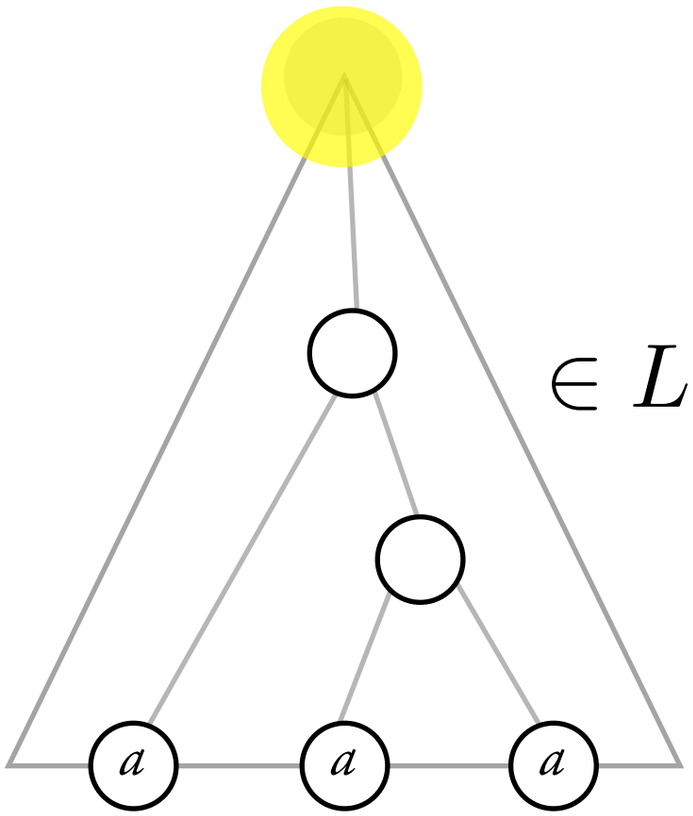

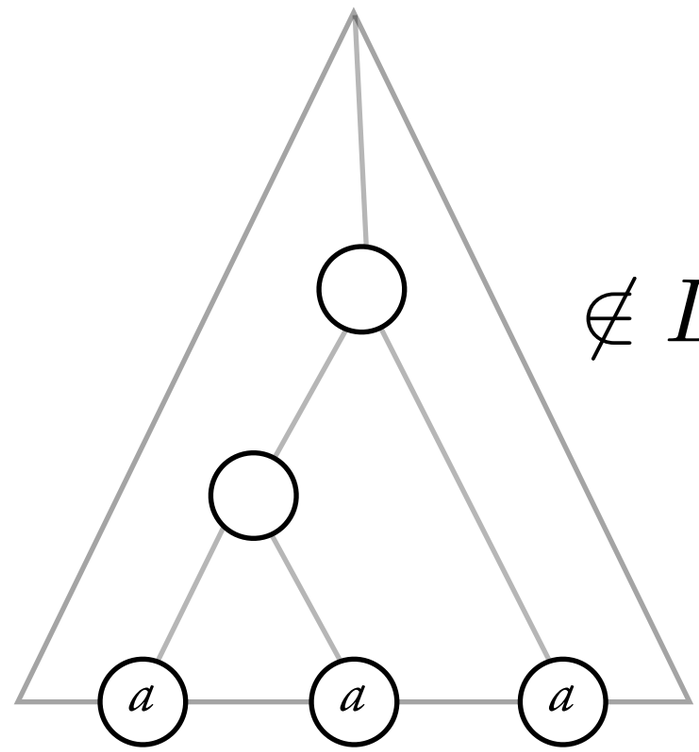

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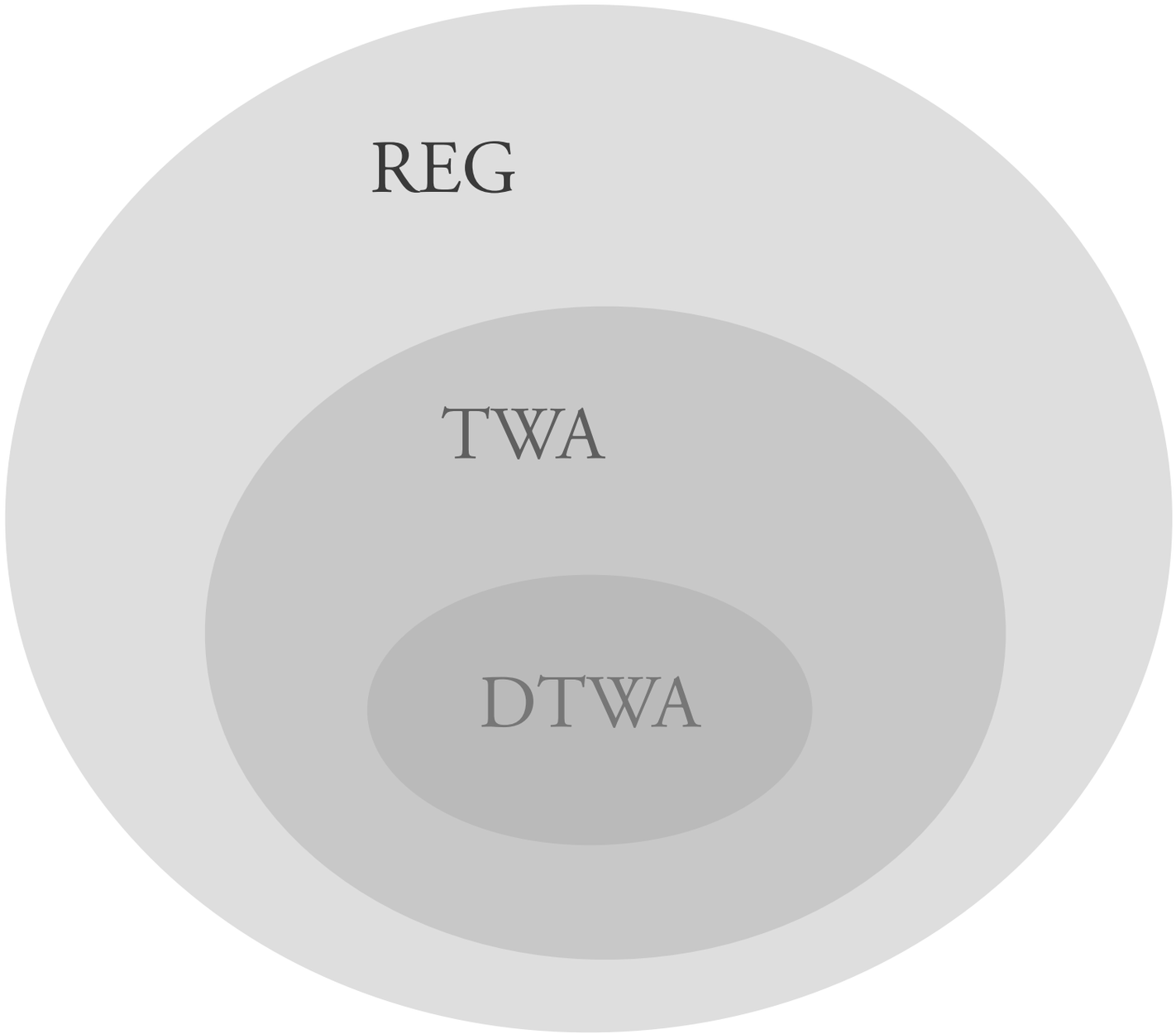
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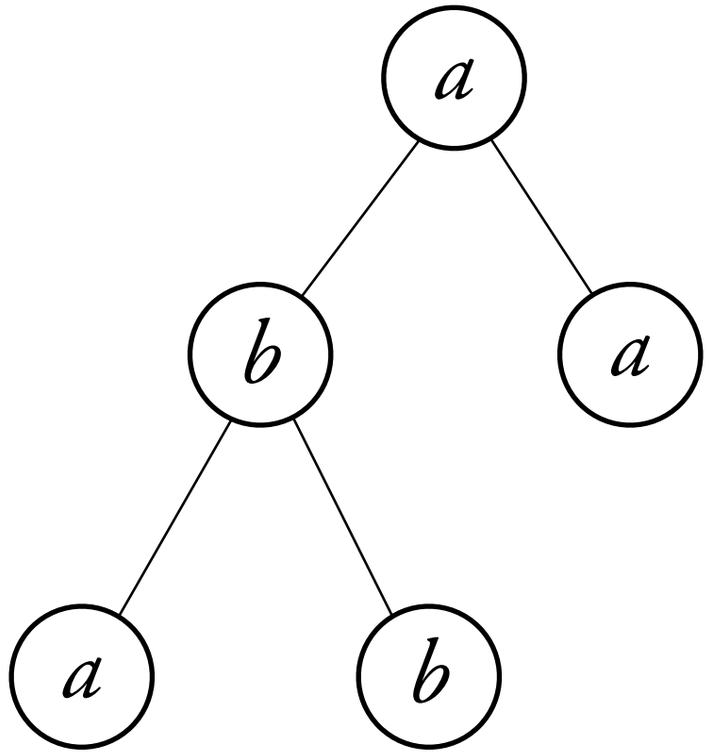

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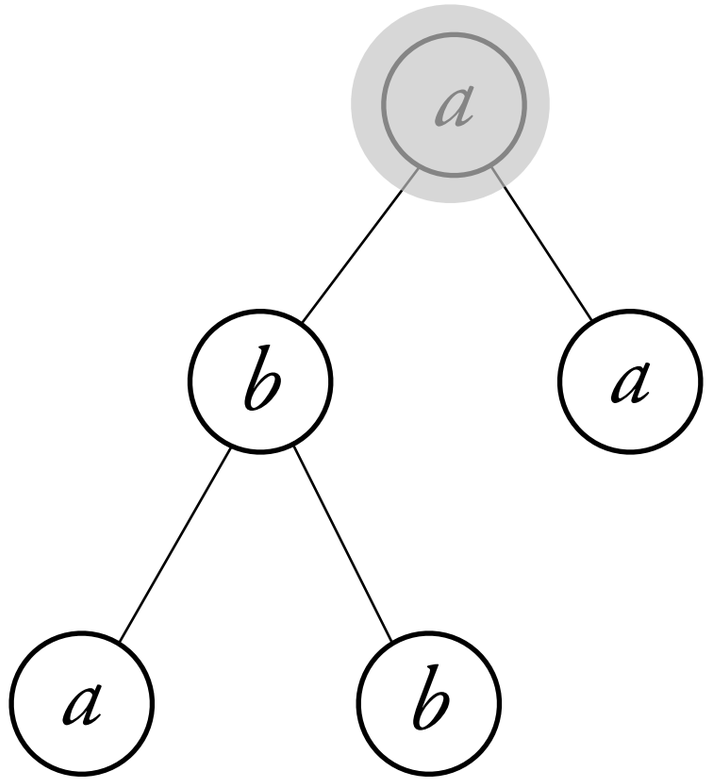

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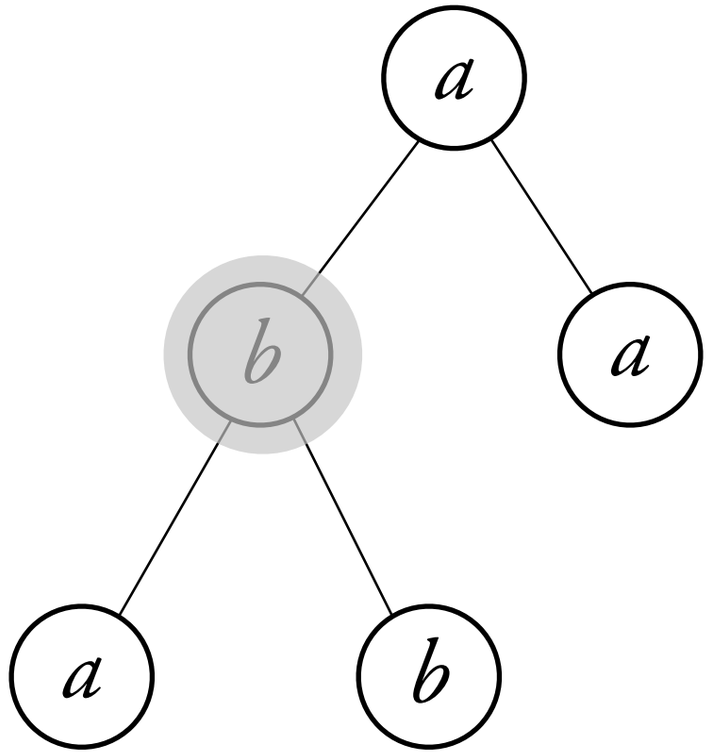

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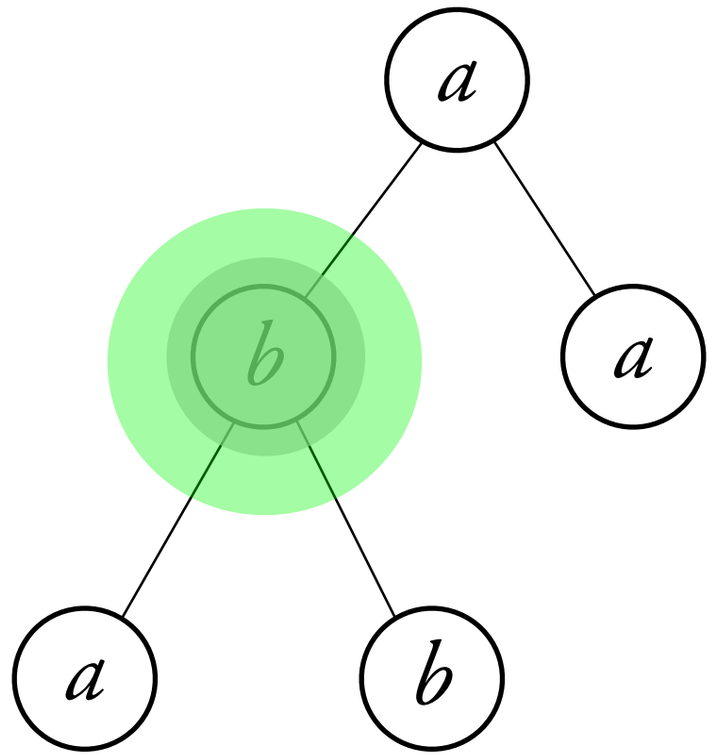

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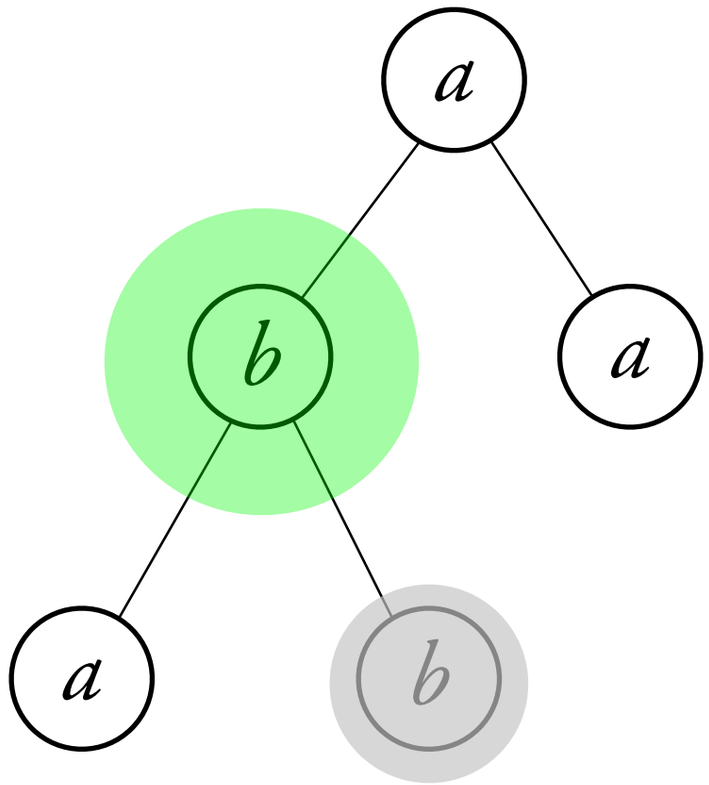

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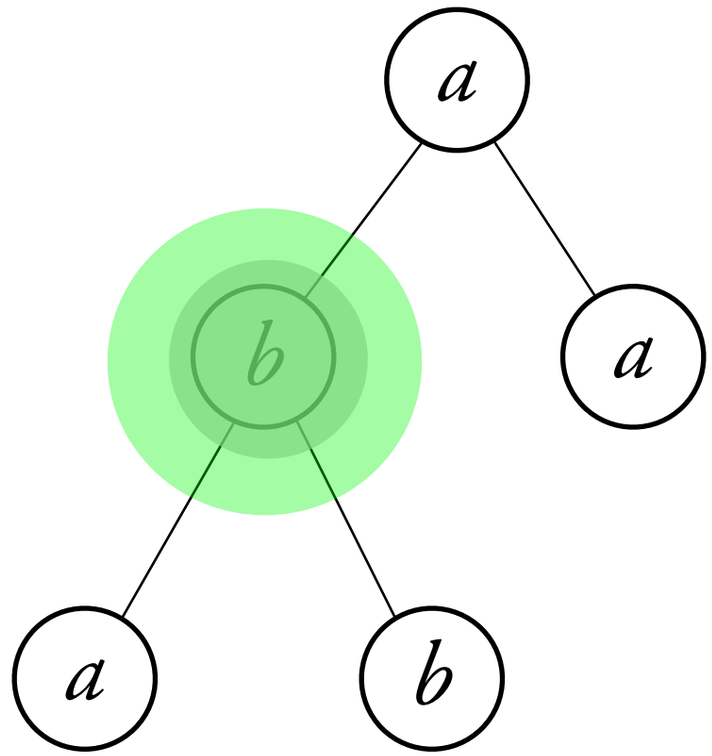

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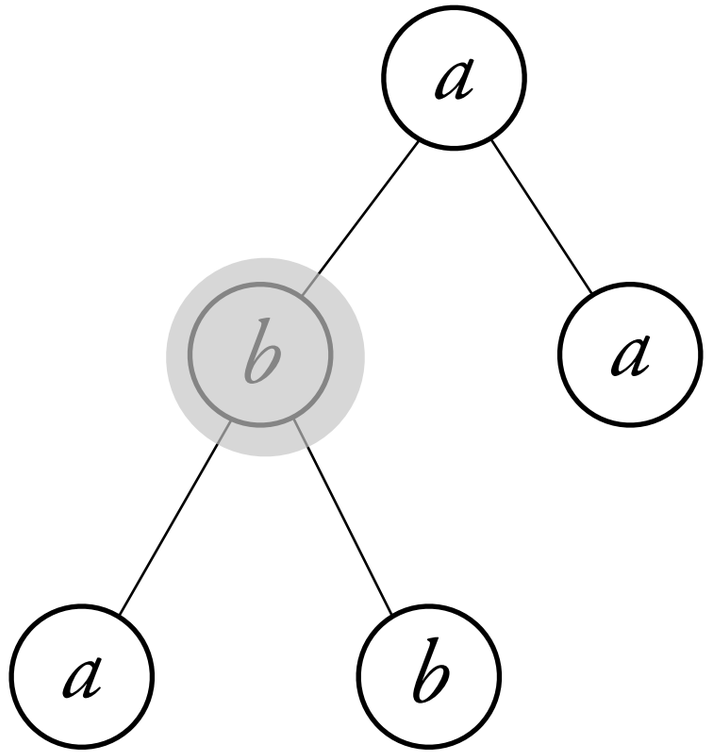

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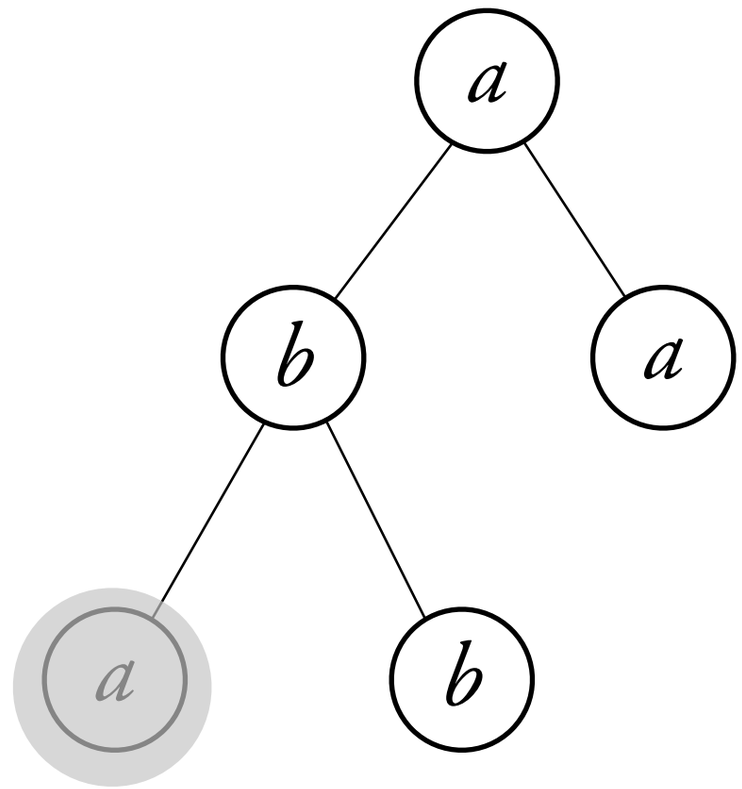

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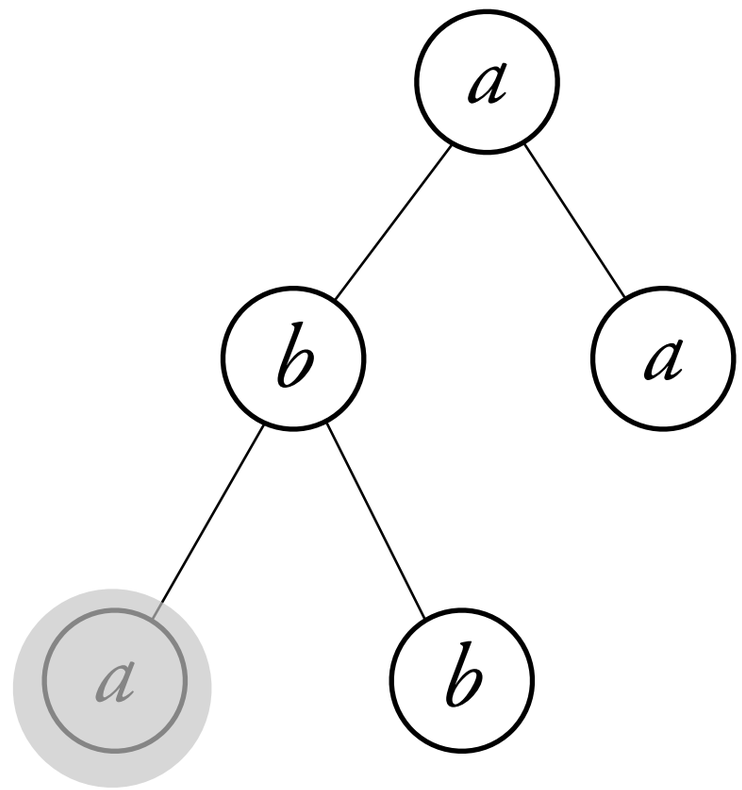

An *n-pebble automaton* has pebbles *1,…,n.*
New tests: “is pebble *i* on the current node?”
New commands: “place pebble *i* on the current node”
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Bad news: emptiness is undecidable for pebble automata,
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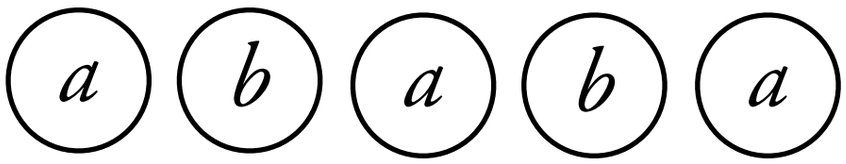

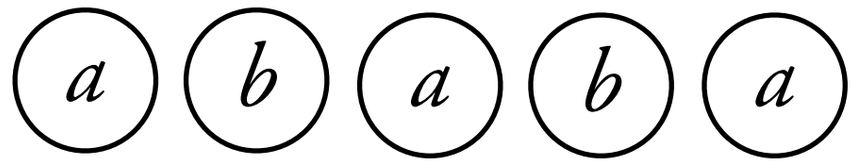

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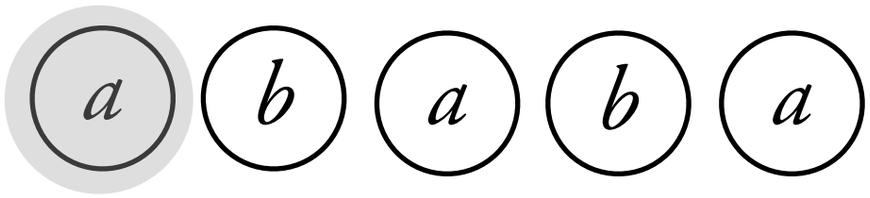

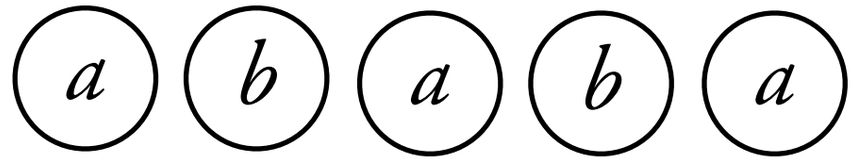

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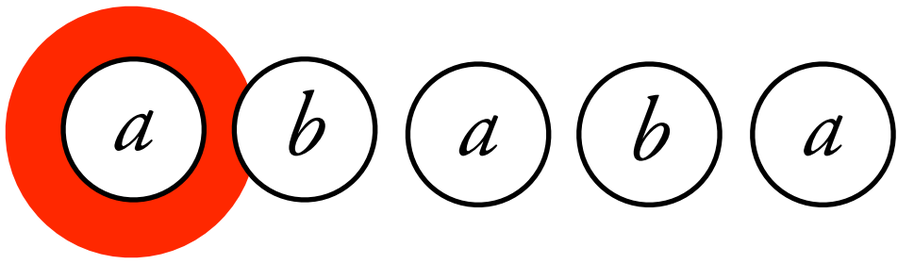

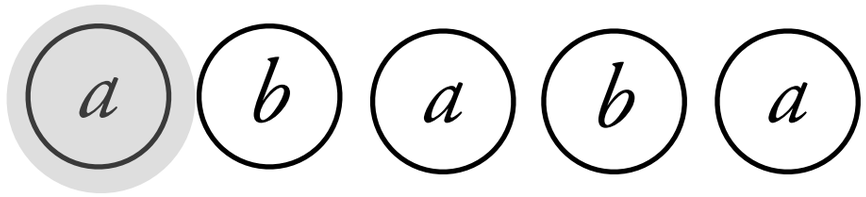

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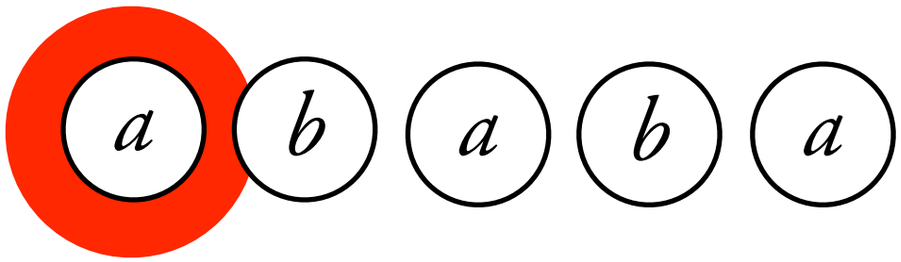

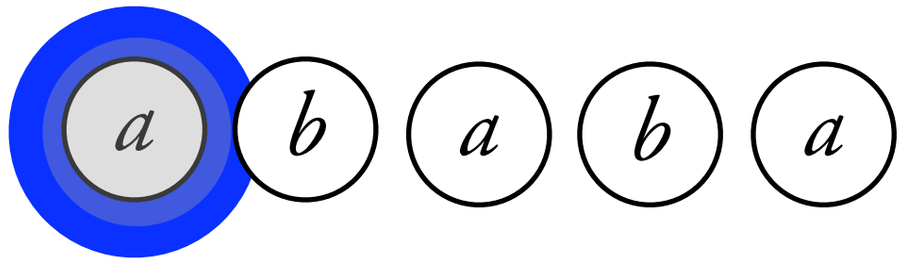

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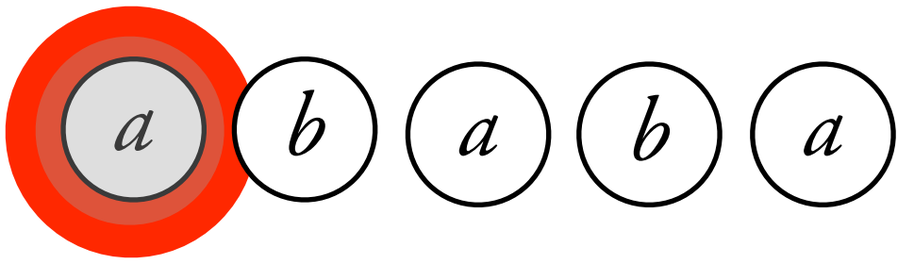

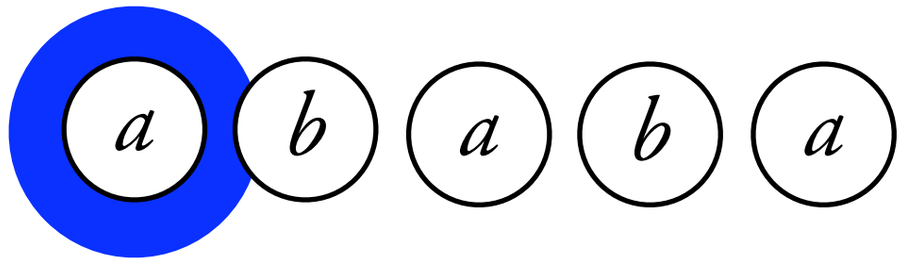

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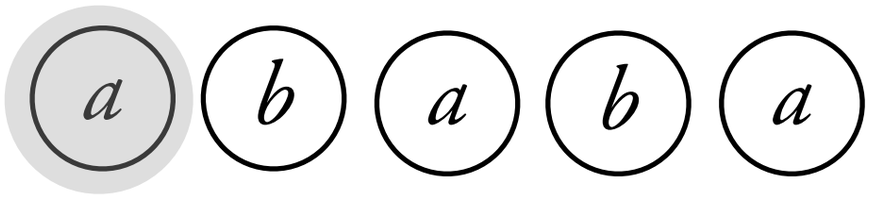

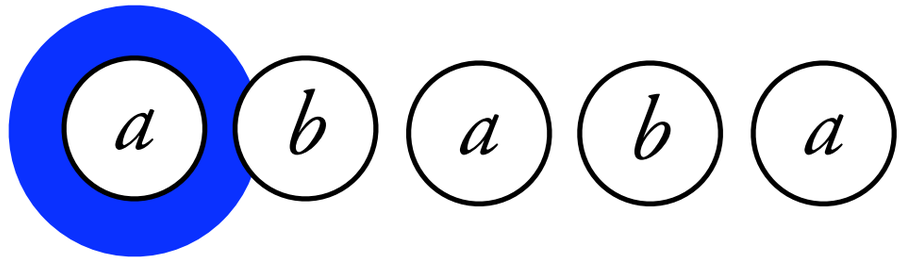

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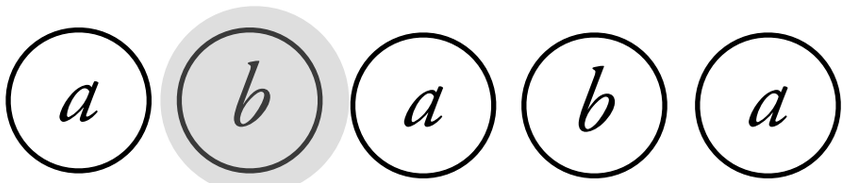

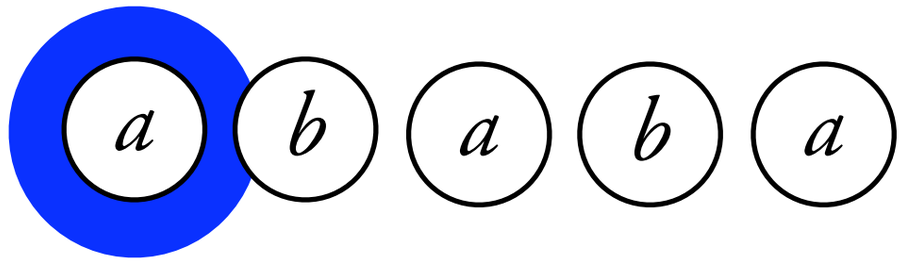

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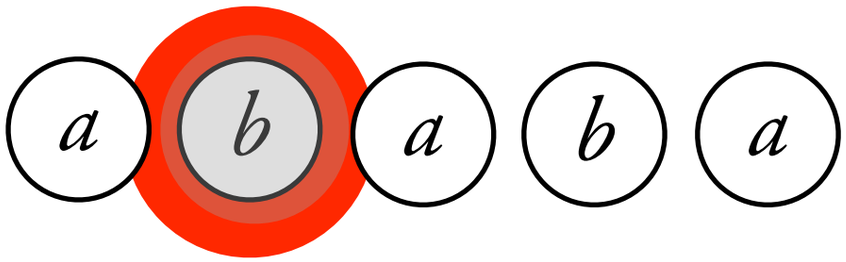

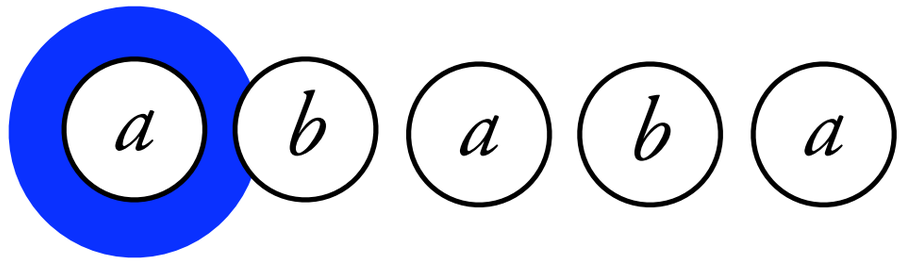

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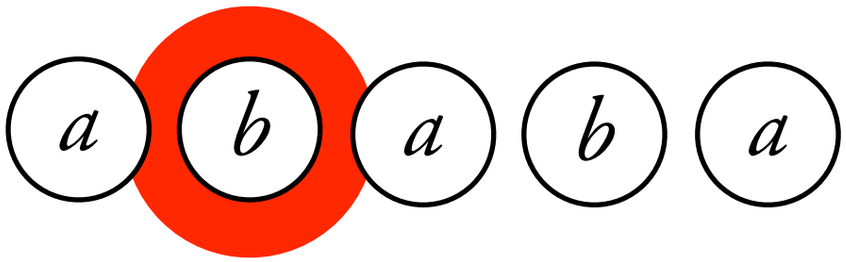

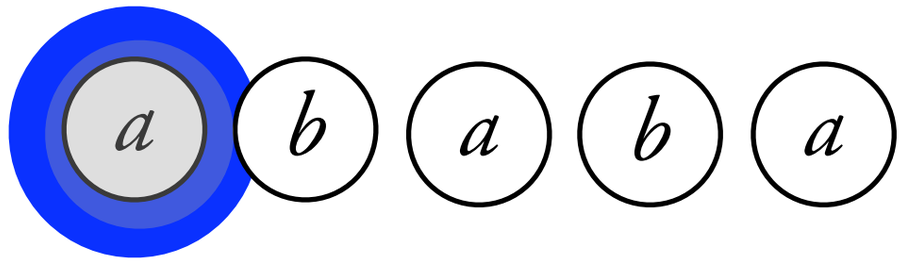

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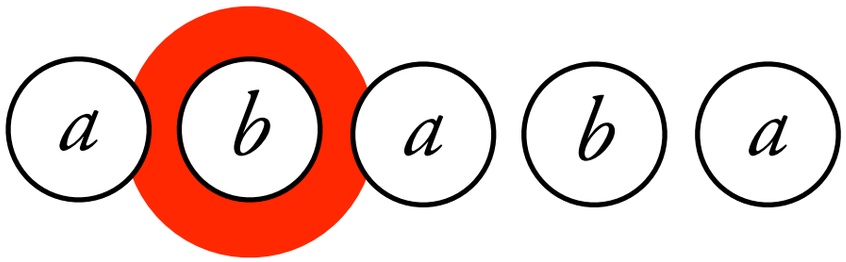

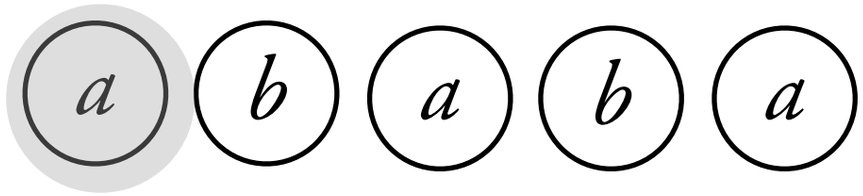

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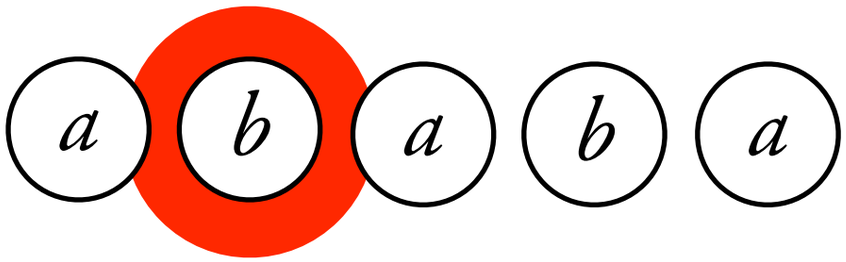

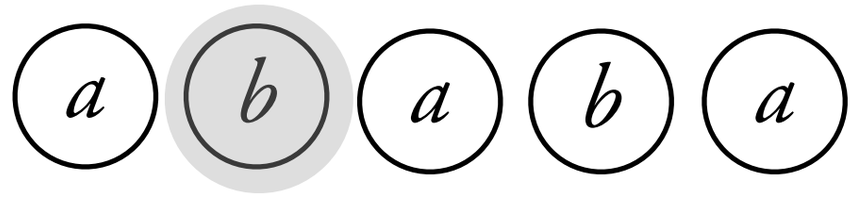

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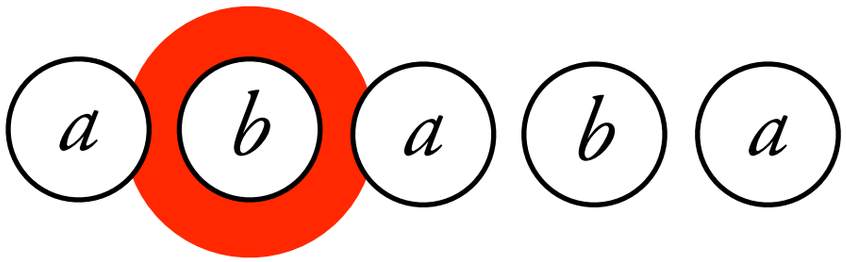

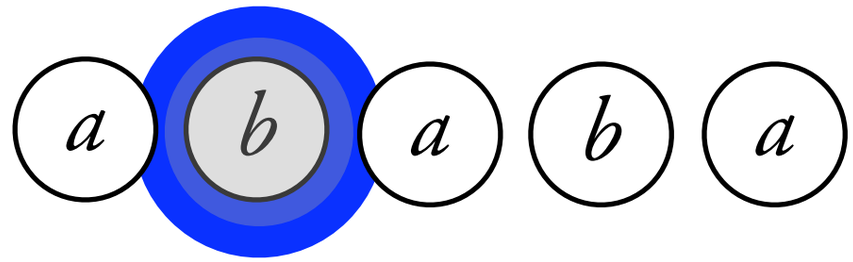

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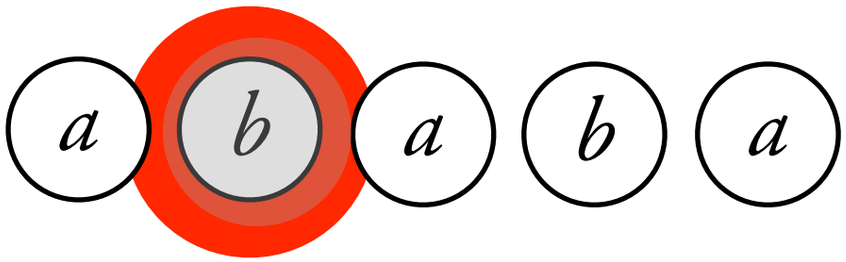

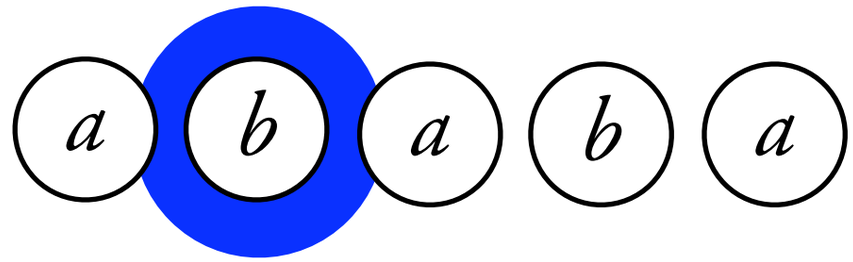

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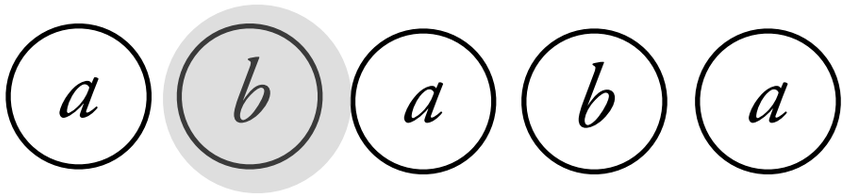

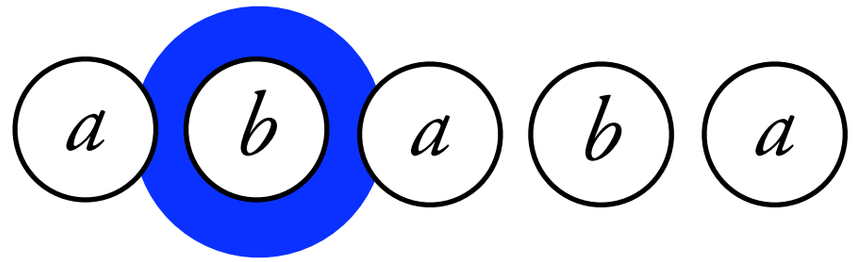

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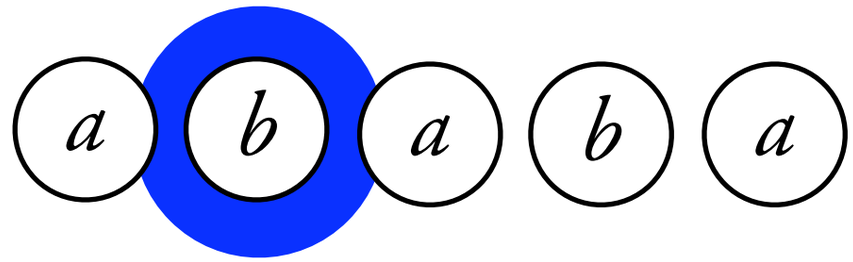

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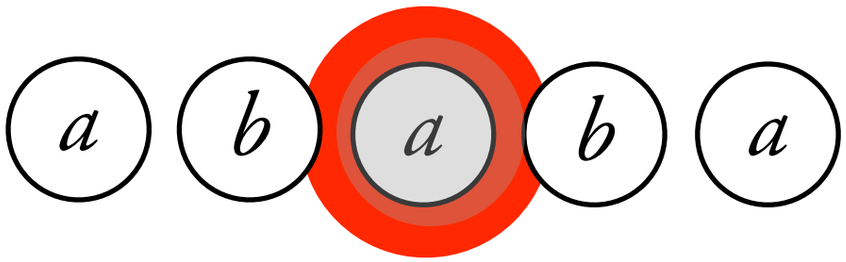

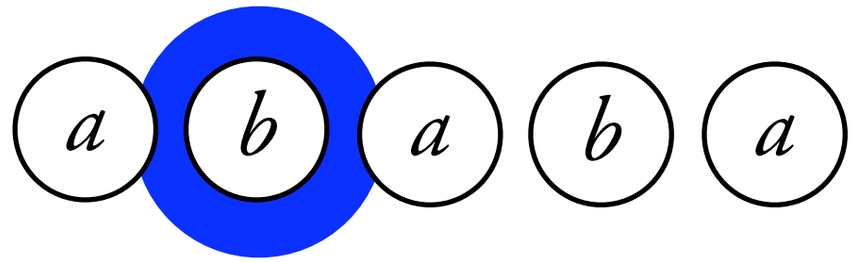

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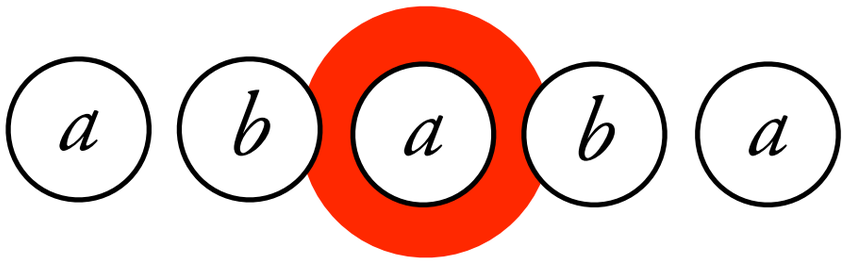

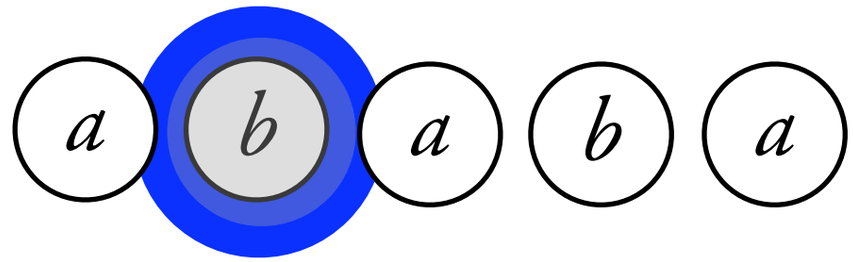

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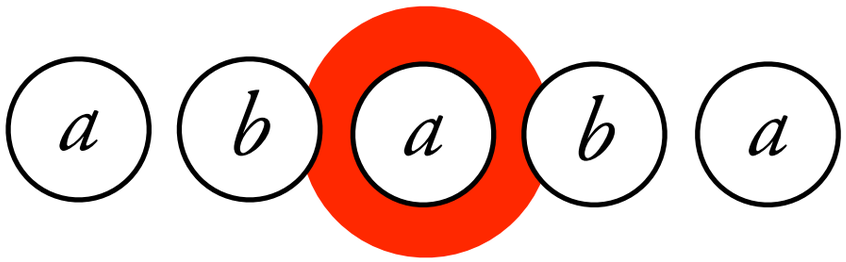

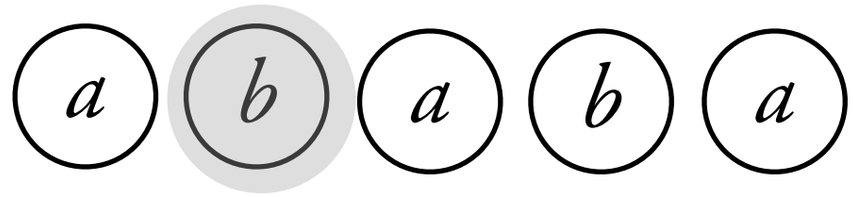

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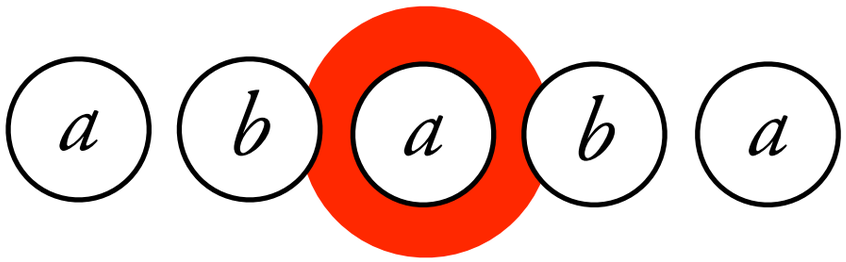

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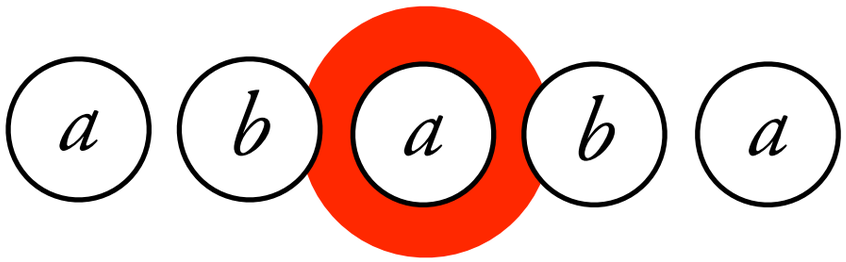

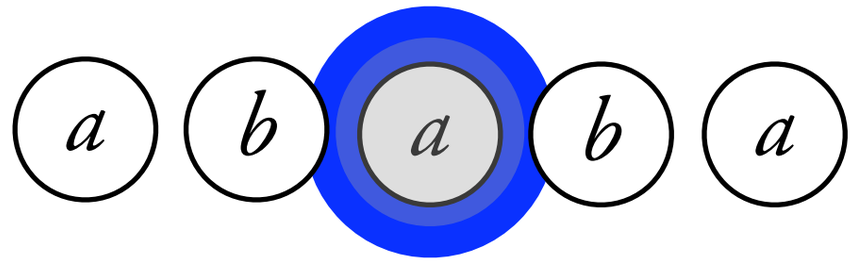

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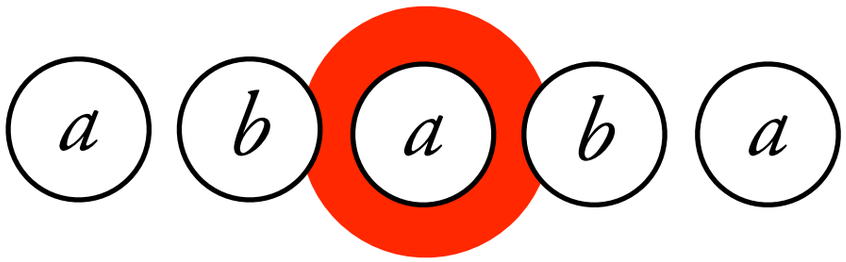

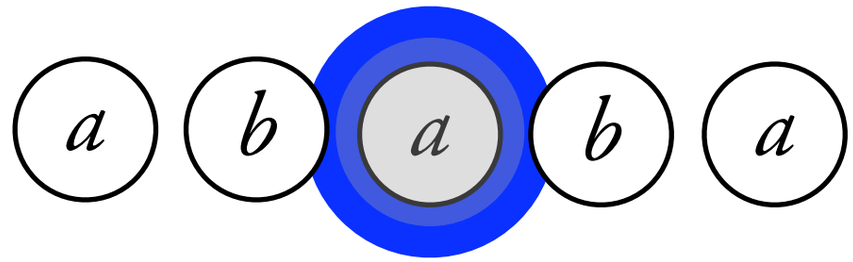

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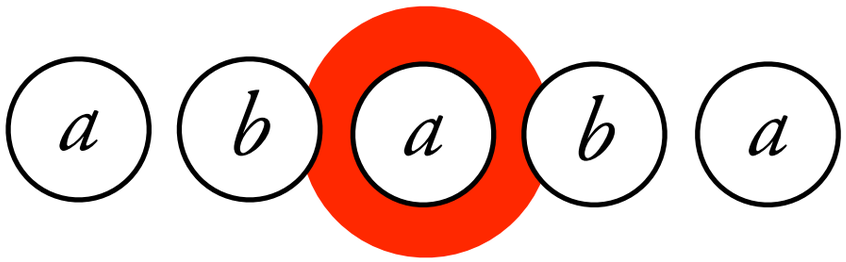

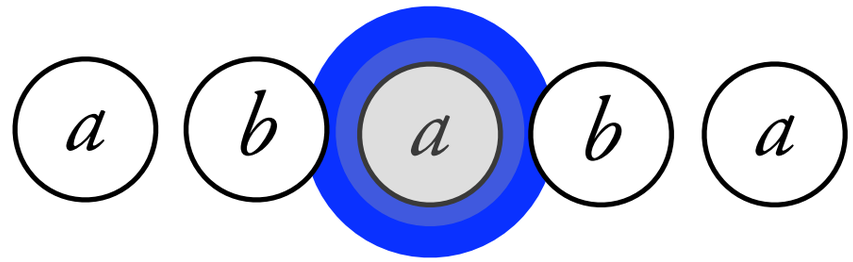

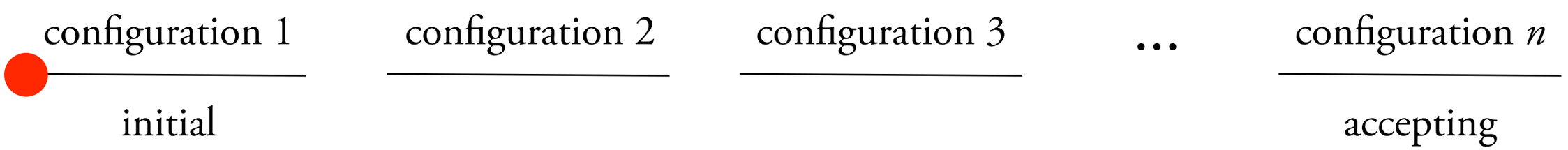

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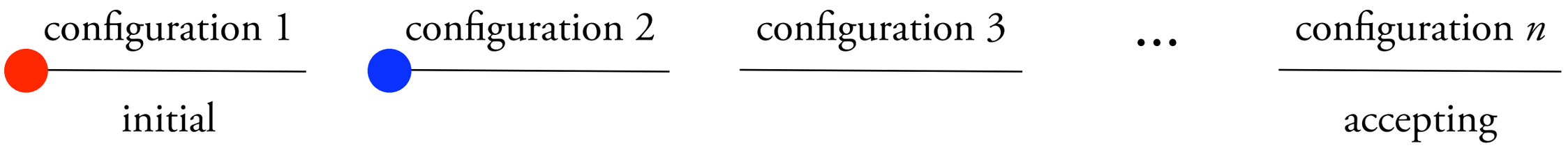

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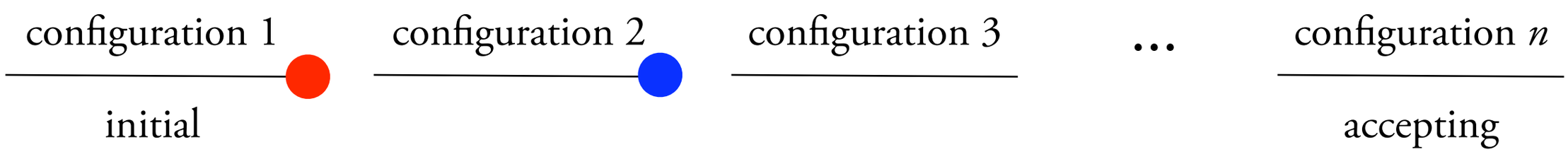

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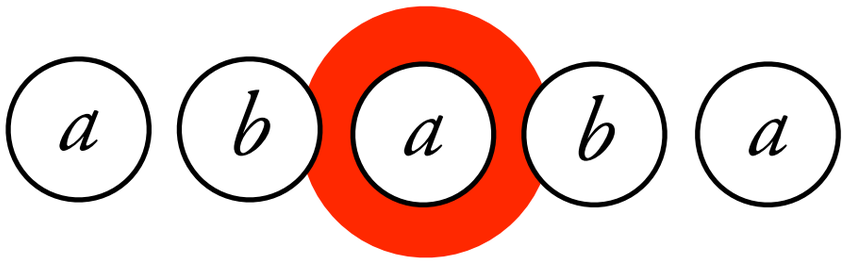

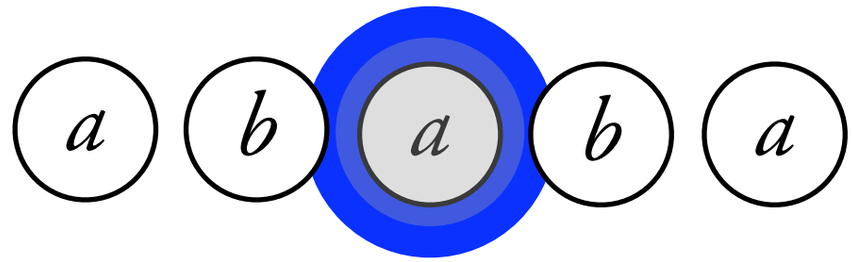

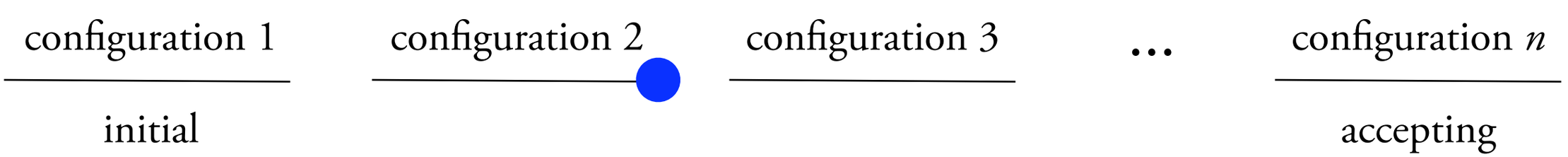

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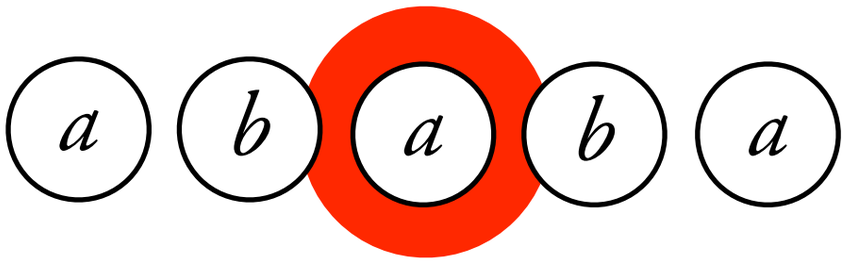

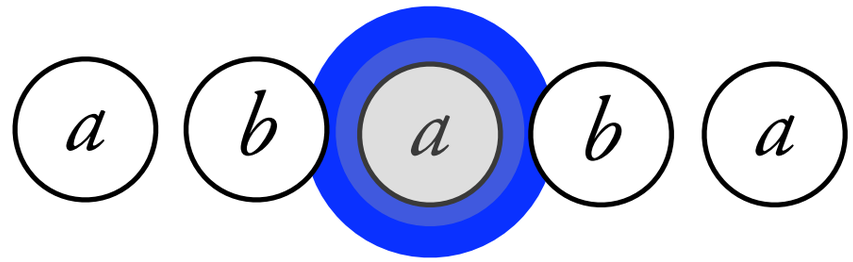

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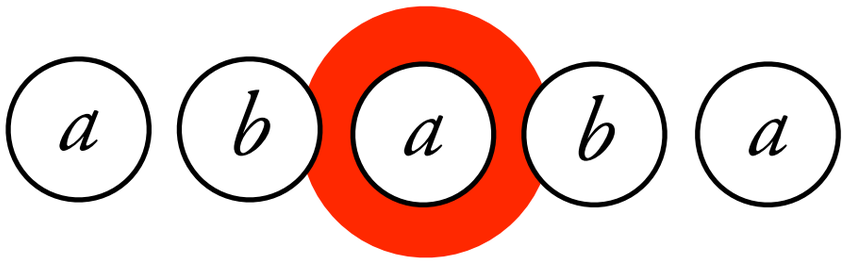

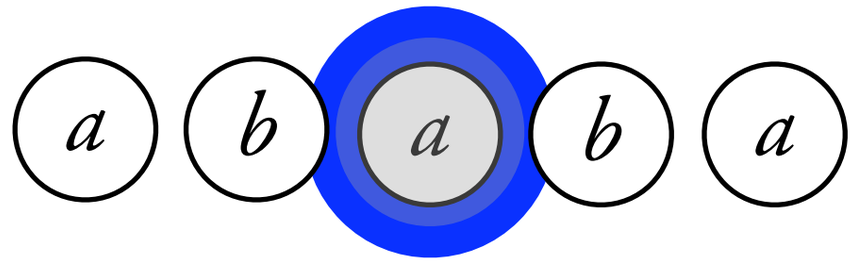

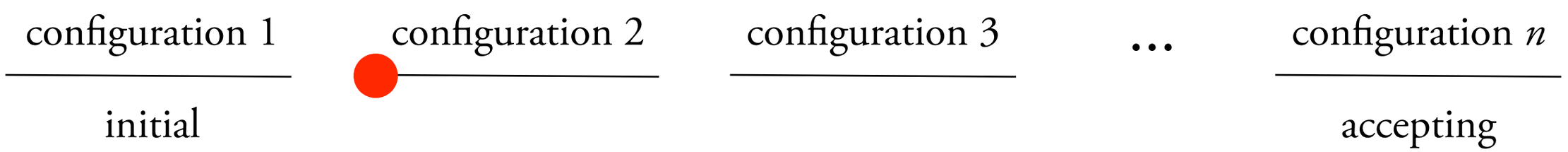

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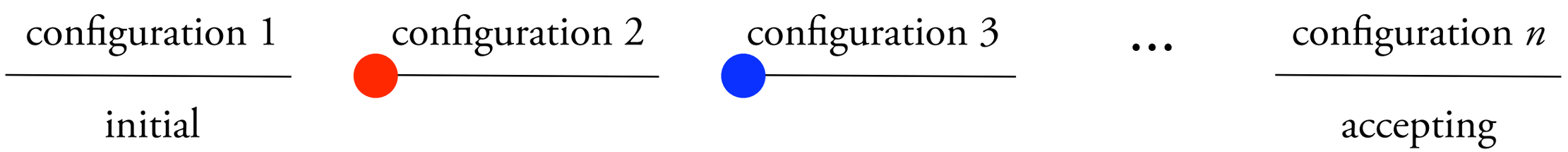

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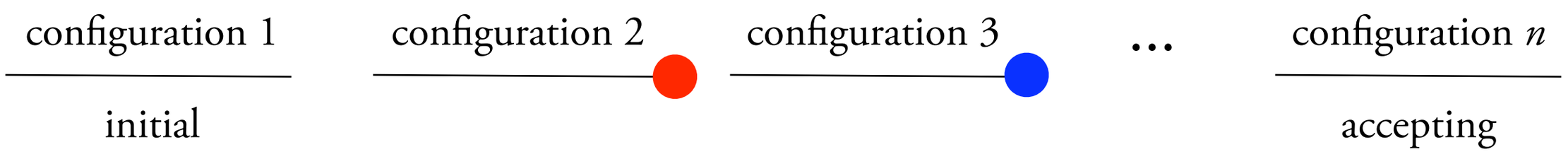

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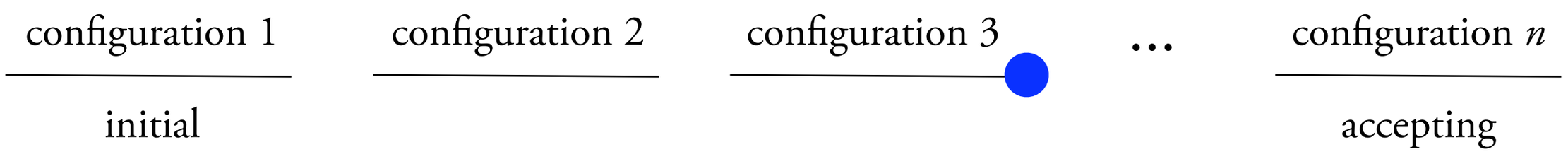

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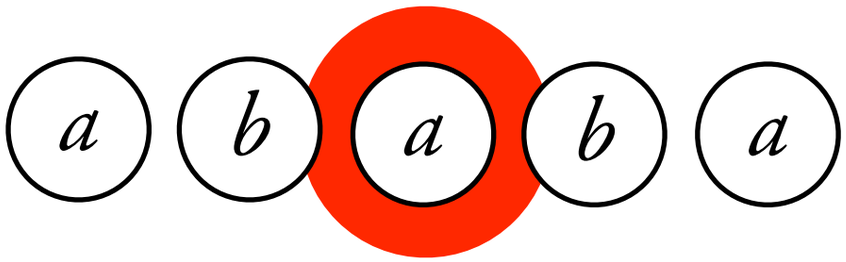

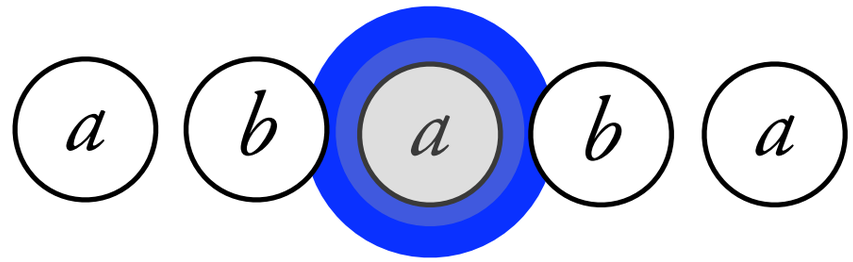

configuration 1	configuration 2	configuration 3	...	configuration n
initial				accepting

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When the newest pebble is *i*, only *i* can be lifted, and *i+1* placed.

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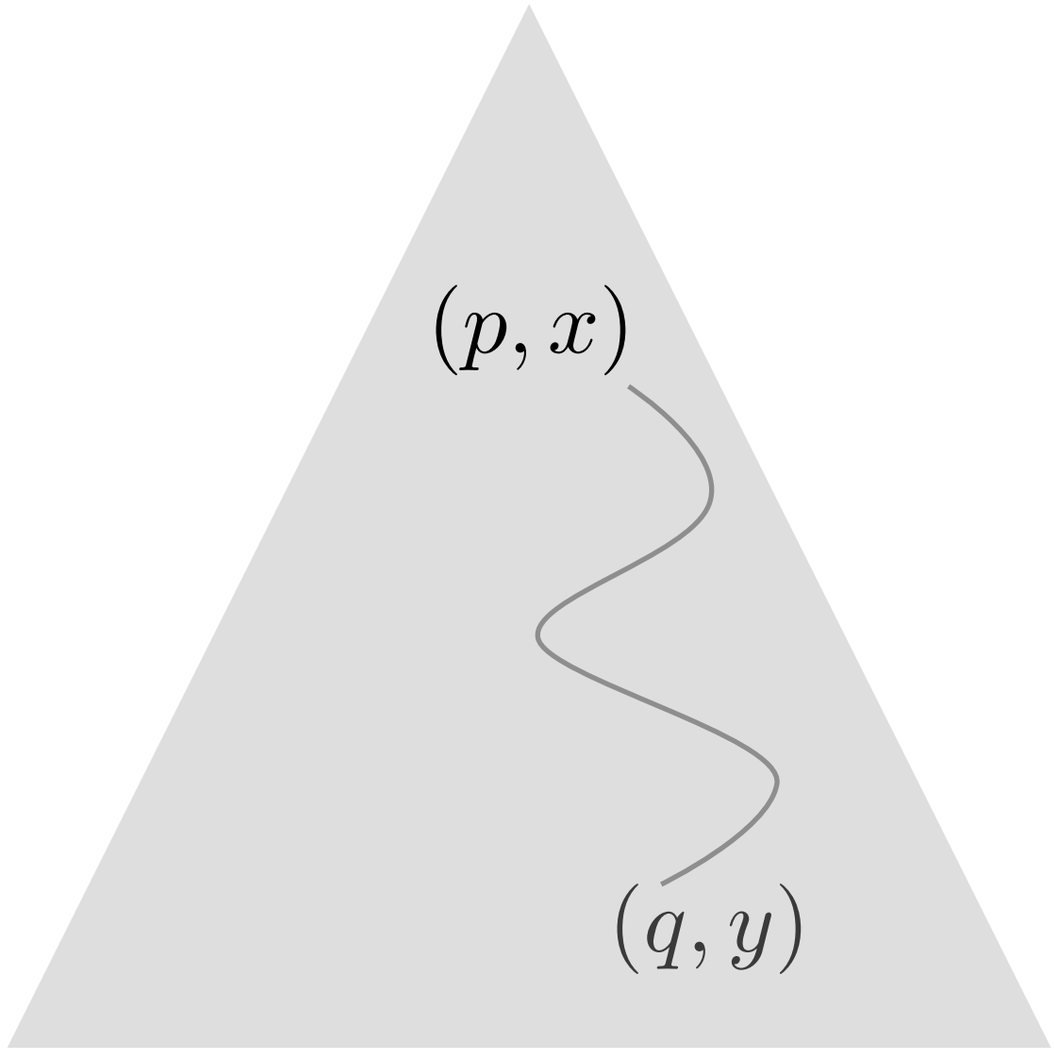
If the pebble automaton has *n* pebbles, the tree automaton may have

$$2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}} \quad n \text{ times}$$

states. Likewise, emptiness is non-elementary.

$$\varphi_{p,q}(x, y, x_1, \ldots, x_i)$$

There is a run that begins in (p, x) and ends in (q, y).
The pebbles at the beginning and end are in $x_1, \ldots, x_i$.
During the run, pebble x_i is not lifted, but pebbles can be added.



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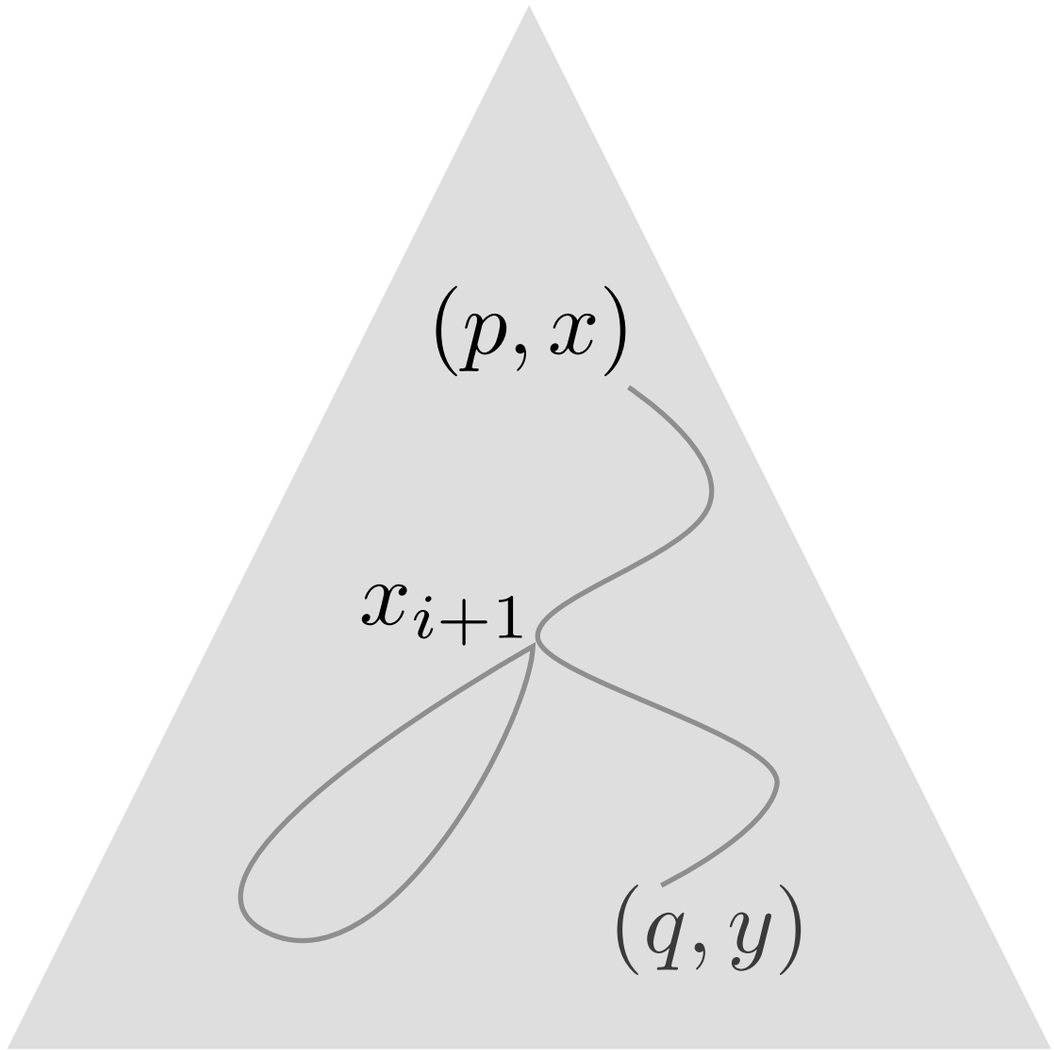

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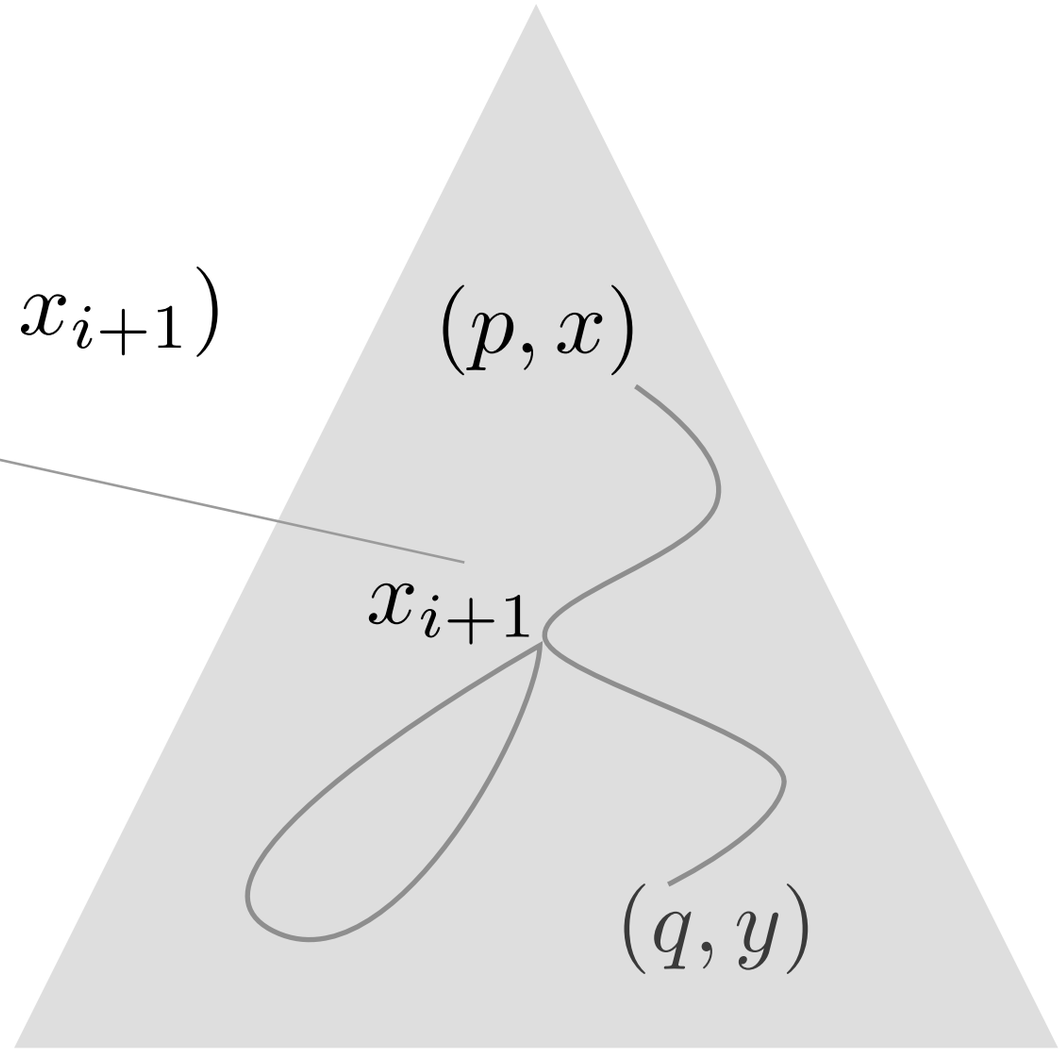
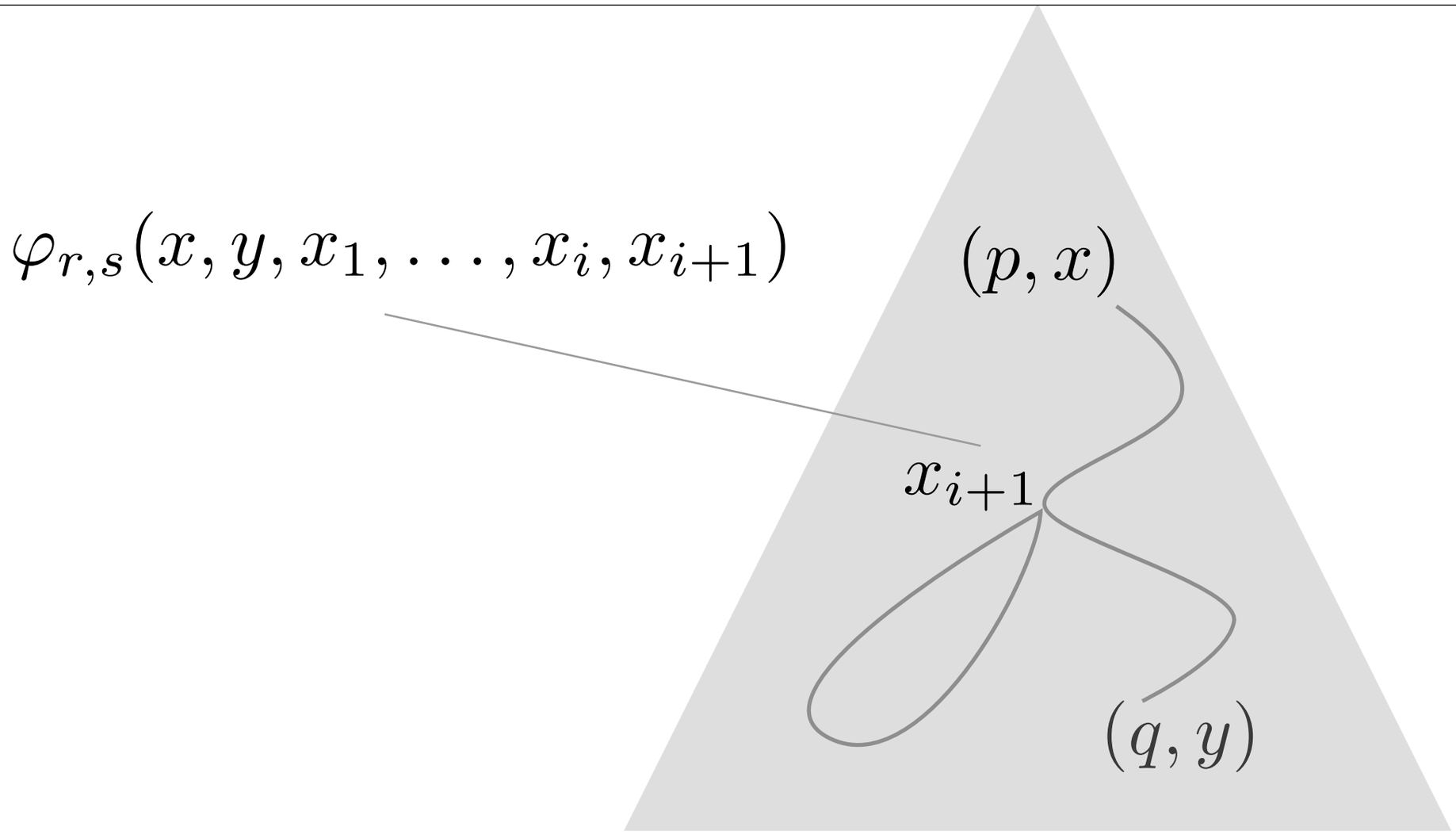
$\varphi_{r,s}(x, y, x_1, \ldots, x_i, x_{i+1})$



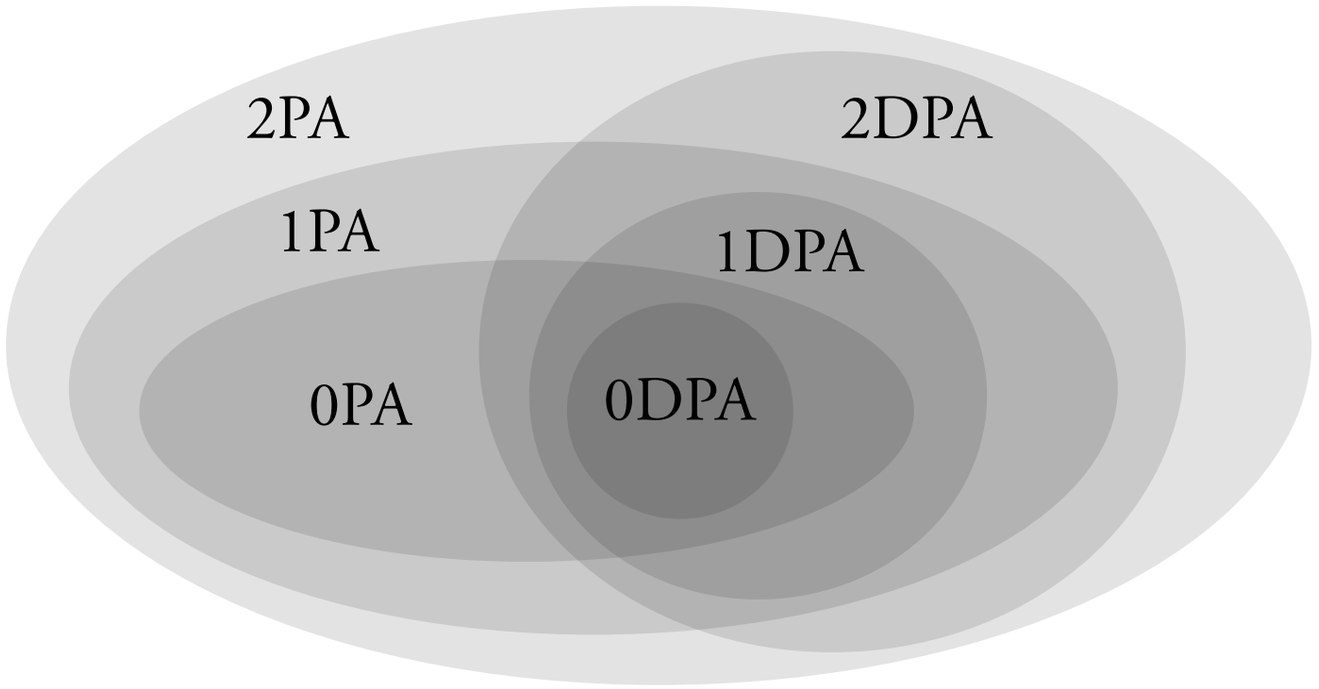
$$\varphi_{p,q}(x, y, x_1, \ldots, x_i)$$

What logic? Monadic second-order logic is good enough.

Pebble automata = first-order logic with positive transitive closure.



Theorem. [B., Samuelides, Schwentick, Segoufin 06]
-Pebble automata do not recognize all regular languages.
-Deterministic *n* pebbles are weaker than nondeterminstic *n* pebbles.
-*n* pebbles are weaker than *n+1* pebbles, both in det and nondet.



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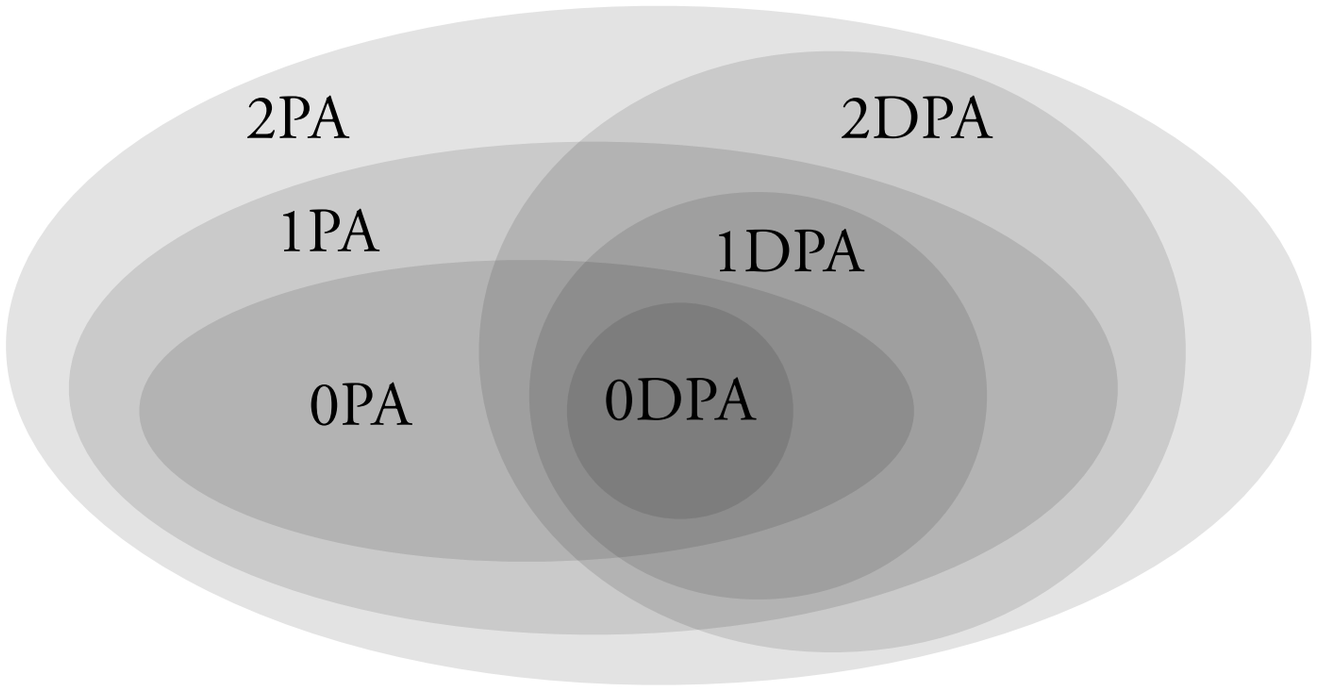

Open question:

$$\bigcup_i i\text{PA} = \bigcup_i i\text{DPA}$$

Known:

$$\forall i \quad 0\text{PA} \not\subseteq i\text{DPA}$$

Pebble automata = first-order logic with positive transitive closure.

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First-order logic.

$$\forall x \forall y \; a(x) \wedge child(x, y) \Rightarrow b(y)$$

For every nodes *x*, *y*, if *x* has label *a* and
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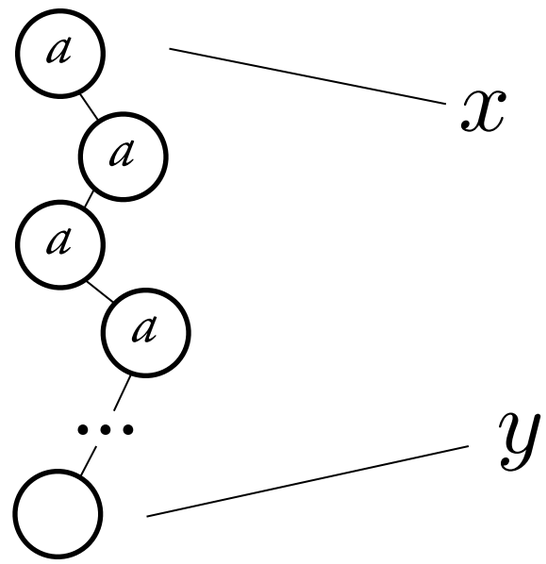
First-order logic with transitive closure.

$$TC\big(child(x, y) \wedge a(x)\big)(x, y)$$



For words,
first-order logic with transitive closure = regular languages.

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What about trees?
first-order logic with positive transitive closure = pebble automata

Theorem. [ten Cate, Segoufin '08]
For trees, not all regular languages are captured by first-order logic with transitive closure.

Conclusion

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Open questions:
-complementation
-detereminization of pebble automata

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Open questions:
-complementation
-detereminization of pebble automata
-better understanding